## WEBVTT

- $1\ 00:00:00.000 \longrightarrow 00:00:03.180$  We want to the last Biostatistics seminar
- $2\ 00:00:03.180 \longrightarrow 00:00:04.740$  for the fall series.
- 3 00:00:04.740 --> 00:00:07.480 It's my great pleasure to welcome our speaker,
- $4~00:00:07.480 \longrightarrow 00:00:09.450~Dr.$  Liangyuan Hu.
- 5~00:00:09.450 --> 00:00:12.680 Dr. Hu is an Assistant Professor of Biostatistics
- $6\ 00:00:12.680 --> 00:00:16.160$  in the Department of Population Health Sciences and Policy
- $7\ 00:00:16.160 --> 00:00:19.050$  at Mount Sinai School of Medicine.
- 8  $00:00:19.050 \longrightarrow 00:00:22.890$  She received her PhD in Biostatistics from Brown University.
- $9\ 00:00:22.890 \longrightarrow 00:00:25.170$  Her methods research focuses on causal inference
- $10\ 00:00:25.170$  --> 00:00:28.280 with complex longitudinal and survival data
- 11 00:00:28.280 --> 00:00:30.150 and Bayesian machine learning.
- $12\ 00:00:30.150 \dashrightarrow 00:00:33.210$  Her independent research has been funded by NIH
- $13~00{:}00{:}33.210 \dashrightarrow 00{:}00{:}36.200$  and Patient Centered Outcomes Research Institute.
- $14\ 00:00:36.200$  --> 00:00:39.194 And her paper in Biometrics has been selected to receive
- $15\ 00:00:39.194$  --> 00:00:44.194 the 2019 Outstanding Statistical Application Award
- $16~00:00:44.880 \longrightarrow 00:00:47.810$  by the American Statistical Association.
- $17\ 00:00:47.810$  --> 00:00:50.270 Today, she's going to share with us her recent work
- $18\ 00:00:50.270 \dashrightarrow 00:00:54.190$  on developing a continuous time marginal structure of models
- $19~00{:}00{:}54.190 \to 00{:}00{:}56.279$  for complex survival outcomes.
- $20\ 00:00:56.279 \longrightarrow 00:00:58.210$  Liangyuan, the floor is yours.
- $21\ 00:00:58.210 \longrightarrow 00:00:59.930$  Well, thank you Li Fan.
- 22 00:00:59.930 --> 00:01:02.079 Thank you so much Fan for your introduction,
- 23 00:01:02.079 --> 00:01:05.300 for the invite also.
- $24\ 00:01:05.300 \longrightarrow 00:01:08.000$  Let me just share my slides full screen.

- 25 00:01:08.000 --> 00:01:10.220 I'm really excited to be here today
- $26\ 00{:}01{:}10.220 \dashrightarrow 00{:}01{:}14.700$  to talk about some of the projects I've been working on
- 27 00:01:14.700 --> 00:01:16.453 in the causal inference field,
- $28\ 00{:}01{:}17.390 \dashrightarrow 00{:}01{:}21.420$  namely, how do we use marginal structure models
- $29\ 00:01:21.420 \longrightarrow 00:01:25.970$  for more complex comparative effectiveness
- $30\ 00:01:25.970 --> 00:01:29.500$  research questions involving continuous-time treatment
- $31\ 00:01:29.500 \longrightarrow 00:01:31.920$  and censored survival outcomes.
- $32\ 00:01:31.920 \longrightarrow 00:01:34.860$  So I'd like to first acknowledge my colleagues,
- 33 00:01:34.860 --> 00:01:37.330 especially Doctors Hogan and Daniels
- $34\ 00:01:37.330 \longrightarrow 00:01:40.170$  who had been instrumental to me
- $35\ 00:01:40.170 \longrightarrow 00:01:42.683$  during the time I was working on this project.
- $36\ 00:01:44.289 --> 00:01:47.453$  And let me just shift to the bar a little if I can.
- 37 00:01:50.120 --> 00:01:51.116 Okay.
- $38\ 00:01:51.116 --> 00:01:56.116$  So this is just for those who aren't very familiar
- $39\ 00:01:56.340 \longrightarrow 00:01:57.540$  with causal inference,
- $40\ 00:01:57.540 --> 00:02:00.570$  and simple slide to introduce the concept.
- $41\ 00:02:00.570 \longrightarrow 00:02:02.020$  Some key concept.
- $42\ 00{:}02{:}02.020\,{\:\mbox{--}}\!>00{:}02{:}06.040$  Suppose we are interested in estimating the causal effect
- 43 00:02:06.040 --> 00:02:10.920 of a binary treatment A on some outcome Y.
- 44 00:02:10.920 --> 00:02:13.340 Using the potential outcomes framework,
- $45\ 00:02:13.340 \longrightarrow 00:02:16.229$  we can define the average treatment effect
- $46\ 00:02:16.229 --> 00:02:19.440$  as the difference between the mean
- $47\ 00:02:19.440 --> 00:02:21.680$  of the two sets of potential outcomes.
- 48 00:02:21.680 --> 00:02:25.620 So, Y1 here is the potential outcome
- $49\ 00:02:25.620 \longrightarrow 00:02:27.030$  that would have been observed
- 50~00:02:27.030 --> 00:02:30.450 had everyone in the population received the treatment.
- 51 00:02:30.450 --> 00:02:32.850 Similarly, Y0 here is the potential outcome

- $52\ 00:02:32.850 \longrightarrow 00:02:34.290$  that would have been observed
- $53\ 00:02:34.290 --> 00:02:37.510$  had no one in the population received the treatment.
- 54 00:02:37.510 --> 00:02:39.490 To estimate the causal effect,
- $55\ 00:02:39.490 -> 00:02:43.450$  the gold standard is the randomized controlled file.
- $56~00:02:43.450 \longrightarrow 00:02:47.820$  So in an RCT, we would randomly allocate patients
- $57\ 00:02:47.820 \longrightarrow 00:02:52.820$  to receive either treatment or the control or placebo,
- $58~00:02:52.870 \dashrightarrow 00:02:56.800$  the randomization would make the two groups of patients
- $59\ 00:02:56.800$  --> 00:03:01.110 more or less very similar in terms of their characteristics.
- $60\ 00:03:01.110 \longrightarrow 00:03:06.110$  So in a sense that these two groups are exchangeable,
- $61\ 00:03:07.600 \longrightarrow 00:03:11.170$  so that an individual's potential outcome
- $62\ 00:03:11.170 \longrightarrow 00:03:13.550$  to either treatment or control
- 63 00:03:13.550 --> 00:03:15.930 would not depend on which treatment group
- $64\ 00:03:15.930 \longrightarrow 00:03:17.810$  this person was assigned to.
- $65\ 00:03:17.810 --> 00:03:20.800$  But just depends on how the treatment works.
- $66~00{:}03{:}20.800 \dashrightarrow 00{:}03{:}23.820$  And this way we can simply look at the difference
- $67\ 00:03:23.820 \longrightarrow 00:03:28.820$  and the mean of the observed outcome
- $68\ 00:03:29.580 \longrightarrow 00:03:31.490$  between the two treatment groups
- $69\ 00:03:31.490 --> 00:03:34.130$  and just to estimate the causal effect.
- 70 00:03:34.130 --> 00:03:35.867 But in many, many situations,
- 71 00:03:35.867 --> 00:03:38.550 we cannot conduct an RCT
- $72\ 00:03:38.550 \longrightarrow 00:03:41.620$  and we have to rely on observational data
- $73~00:03:41.620 \longrightarrow 00:03:44.680$  to get the causal inference about treatment effects.
- $74\ 00:03:44.680 \longrightarrow 00:03:46.530$  So in these situations,
- $75~00{:}03{:}46.530 \dashrightarrow 00{:}03{:}49.140$  the independence between the potential outcome

- $76~00{:}03{:}49.140$  -->  $00{:}03{:}51.823$  and treatment assignment would no longer hold.
- $77\ 00:03:52.870 \longrightarrow 00:03:55.660$  Because there might be exists a confounder
- $78\ 00:03:55.660 --> 00:03:58.240$  that is predictive of the outcome,
- 79~00:03:58.240 --> 00:04:01.210 such that the probability of receiving the treatment
- $80\ 00:04:01.210 \longrightarrow 00:04:02.640$  depends on the confounder.
- $81\ 00:04:02.640 \longrightarrow 00:04:06.093$  So for example, age might be such a confounder.
- $82\ 00:04:06.093 \dashrightarrow 00:04:09.900$  For example, younger patients may be more likely
- 83  $00:04:09.900 \longrightarrow 00:04:12.620$  to receive the treatment.
- 84 00:04:12.620 --> 00:04:13.490 So in this case,
- $85\ 00:04:13.490 --> 00:04:16.410$  if you take the difference in the average
- $86~00:04:16.410 \longrightarrow 00:04:19.600$  of the observed outcome between the two groups,
- $87\ 00{:}04{:}19.600 \dashrightarrow 00{:}04{:}23.880$  then this estimate would not bear a causal interpretation
- $88\ 00:04:23.880 --> 00:04:27.173$  because the difference might be confounded by age.
- 89 00:04:28.630 --> 00:04:31.100 So we would have to use specialized
- 90 00:04:31.100 --> 00:04:34.500 causal inference techniques to remove the confounding.
- 91 00:04:34.500 --> 00:04:37.420 And there are just many, many techniques out there.
- $92\ 00:04:37.420$  --> 00:04:41.282 but today I'm just gonna focus on marginal structure model,
- 93 00:04:41.282 --> 00:04:46.150 because it is simple to implement.
- 94 00:04:46.150 --> 00:04:48.960 It has good statistical properties,
- $95\ 00:04:48.960 \longrightarrow 00:04:50.740$  and it is versatile enough
- $96\ 00:04:50.740 \longrightarrow 00:04:53.550$  to accommodate many, many complications
- $97\ 00:04:53.550 --> 00:04:57.520$  posed by observational data that I'll talk about later.
- $98~00:04:57.520 \longrightarrow 00:05:00.170$  So we can propose a marginal structure model

- $99\ 00:05:00.170 --> 00:05:04.410$  relating the potential outcome to the treatment assignment.
- $100\ 00{:}05{:}04.410 --> 00{:}05{:}08.020$  And here theta one would capture the causal effect.
- 101 00:05:08.020 --> 00:05:09.070 But in reality,
- $102\ 00:05:09.070 --> 00:05:12.040$  we can only fit a model to the observer data.
- $103\ 00:05:12.040 \longrightarrow 00:05:15.611$  And as I talked earlier,
- 104 00:05:15.611 --> 00:05:19.110 the parameter estimator beta one here
- 105 00:05:19.110 --> 00:05:23.270 would not bear a causal interpretation,
- $106\ 00:05:23.270 --> 00:05:25.600$  it just measures association.
- $107\ 00:05:25.600 \longrightarrow 00:05:27.840$  But we can get to causation
- $108\ 00:05:27.840 \longrightarrow 00:05:32.280$  if by solving the weighted estimating equation,
- $109\ 00{:}05{:}32.280 \dashrightarrow 00{:}05{:}37.280$  using the weight as W inverse of conditional probability
- $110\ 00{:}05{:}39.620 \dashrightarrow 00{:}05{:}43.380$  of treatment assignment given the measured covariance.
- 111 00:05:43.380 --> 00:05:45.560 And this works because the IP weighting
- $112\ 00{:}05{:}45.560 {\: \hbox{--}}{>}\ 00{:}05{:}47.430$  or inverse probability weighting
- 113 00:05:47.430 --> 00:05:51.350 removes confounding by measured covariance X
- $114\ 00:05:51.350 \longrightarrow 00:05:53.373$  in the weighted pseudo-population.
- $115\ 00:05:54.220 --> 00:05:57.433$  So that's just a simple example
- 116  $00:05:57.433 \longrightarrow 00:06:02.433$  to illustrate the use of marginal structure model.
- 117 00:06:02.880 --> 00:06:05.880 And traditionally treatment assignment,
- $118\ 00:06:05.880 \dashrightarrow 00:06:09.760$  treatment is assigned at baseline and it's time fixed.
- $119\ 00:06:09.760 \dashrightarrow 00:06:13.663$  So it means that the treatment doesn't change over time,
- $120\ 00{:}06{:}13.663 \dashrightarrow 00{:}06{:}18.663$  but with increased availability of healthcare data sets,
- $121\ 00:06:20.910 \longrightarrow 00:06:23.490$  there are increased demands for more refined
- $122\ 00:06:23.490 \longrightarrow 00:06:27.590$  causal inference methods to evaluate complex
- $123\ 00{:}06{:}27.590 --> 00{:}06{:}28.930\ {\rm treatment\ regimens}.$

- $124\ 00:06:28.930 \longrightarrow 00:06:32.817$  So one example is that treatment initiation
- $125\ 00:06:34.010$  --> 00:06:38.673 can actually depend on time, so it changes over time.
- $126\ 00:06:39.981 \longrightarrow 00:06:40.814$  In this case,
- 127 00:06:40.814 --> 00:06:43.930 it would just be impractical to conduct RCTs
- $128\ 00:06:43.930 \longrightarrow 00:06:45.590$  because there are just simply too many
- 129 00:06:45.590 --> 00:06:47.323 treatment initiation time points.
- $130\ 00{:}06{:}48.290 \dashrightarrow 00{:}06{:}51.963$  So I'm going to use two motivating examples in this talk.
- $131\ 00:06:53.920 \dashrightarrow 00:06:58.920$  The first example is about timing of treatment initiation
- $132\ 00{:}06{:}59.880$  -->  $00{:}07{:}04.880$  for patients who present both HIV and TB, tuberculosis.
- $133\ 00:07:05.130 \longrightarrow 00:07:06.570$  For these patients,
- 134 00:07:06.570 --> 00:07:09.380 TB treatment will be initiated immediately
- $135\ 00{:}07{:}09.380 \dashrightarrow 00{:}07{:}13.120$  after the diagnosis, but during the TB treatment,
- $136\ 00{:}07{:}13.120 \dashrightarrow 00{:}07{:}17.260$  when is the optimal time to initiate the HIV treatment
- 137 00:07:17.260 --> 00:07:19.620 or ART, anti-retroviral therapy?
- $138\ 00:07:19.620 \longrightarrow 00:07:22.430$  That is a very important question to answer,
- $139\ 00:07:22.430 \longrightarrow 00:07:25.010$  because if you initiate the treatment too early,
- 140 00:07:25.010 --> 00:07:28.610 there might be drug interactions, drug toxicity,
- 141 00:07:28.610 --> 00:07:30.870 but if you delay the treatment too much,
- $142\ 00:07:30.870 \longrightarrow 00:07:33.050$  then there's also increased the mortality
- $143\ 00:07:33.050 \longrightarrow 00:07:34.913$  associated with AIDS.
- $144\ 00{:}07{:}35.980 \dashrightarrow 00{:}07{:}39.370$  The second example is timing of HIV treatment
- $145\ 00:07:39.370 \longrightarrow 00:07:40.543$  for adolescents.
- 146 00:07:41.830 --> 00:07:44.750 The timing now is defined with respect
- $147\ 00:07:44.750 \longrightarrow 00:07:48.577$  to the evolving value of a biomarker CD4.
- $148\ 00{:}07{:}48.577 \dashrightarrow 00{:}07{:}51.430$  And this is also an important question to answer

- $149\ 00:07:51.430 --> 00:07:56.430$  because the WHO guideline is in the form of
- $150~00{:}07{:}57.130 \dashrightarrow 00{:}08{:}01.350$  treat this person when the person's CD4 cell count
- 151 00:08:01.350 --> 00:08:04.150 drops below 350, for example,
- $152\ 00:08:04.150 \longrightarrow 00:08:06.450$  and for the population of adolescents
- 153 00:08:06.450 --> 00:08:10.440 currently there's no concrete evidence
- $154\ 00:08:10.440 --> 00:08:13.753$  for supporting the optimal threshold.
- $155\ 00:08:15.370 \longrightarrow 00:08:20.040$  So to statistically formulate these two motivating examples,
- $156\ 00:08:20.040 \longrightarrow 00:08:21.380$  the first one,
- $157\ 00:08:21.380 --> 00:08:24.720$  when is the best time to initiate a treatment?
- $158\ 00:08:24.720 \longrightarrow 00:08:27.120$  So this is actually a static treatment regimen
- 159 00:08:27.120 --> 00:08:28.703 with respect to time,
- $160\ 00:08:31.919 --> 00:08:32.752$  and the initiation can occur on the continuous timescale.
- $161\ 00{:}08{:}34.670 \dashrightarrow 00{:}08{:}38.690$  And second example is actually a dynamic treatment regimen.
- $162\ 00{:}08{:}38.690 \dashrightarrow 00{:}08{:}43.690$  It's dynamic because it depends on the evolving history
- 163 00:08:44.890 --> 00:08:47.528 of treatment and a biomarker,
- $164\ 00:08:47.528$  --> 00:08:52.528 but initiation can also occur on the continuous timescale.
- $165\ 00:08:53.570 \longrightarrow 00:08:56.380$  So marginal structure models are suitable
- 166 00:08:56.380 --> 00:08:59.300 for addressing a time dependent treatment,
- 167 00:08:59.300 --> 00:09:02.000 but in order to use the models,
- $168\ 00:09:02.000$  --> 00:09:04.980 we have to overcome some statistical challenges.
- $169\ 00:09:04.980 \longrightarrow 00:09:07.530$  The first challenge is that we need
- $170\ 00:09:07.530 \longrightarrow 00:09:11.070$  to estimate the causal effect of the actual timing,
- $171\ 00:09:11.070 --> 00:09:15.360$  not compare protocols defined by some specific intervals,
- $172\ 00:09:15.360 \longrightarrow 00:09:18.653$  which is a lot of existing studies did.

- 173 00:09:21.331  $\rightarrow$  00:09:25.140 And also a lot of RCT reported these kinds of results.
- 174 00:09:25.140 --> 00:09:26.690 Because as I said earlier,
- $175\ 00:09:26.690 \longrightarrow 00:09:29.700$  it's just impractical for RCTs
- $176\ 00:09:29.700 \longrightarrow 00:09:32.713$  to report continuous time causal effects.
- $177\ 00:09:33.620 \longrightarrow 00:09:36.320$  We would also need to address complications
- $178\ 00:09:36.320 \longrightarrow 00:09:38.840$  posed by observational data.
- $179\ 00:09:38.840 \longrightarrow 00:09:41.240$  This is something I'll talk about later.
- $180\ 00:09:41.240 \dashrightarrow 00:09:44.750$  And also we are dealing with censored survival outcomes
- 181 00:09:44.750 --> 00:09:47.393 that adds another layer of complexity.
- $182\ 00{:}09{:}48.550 \dashrightarrow 00{:}09{:}53.300$  So these are four sensory patterns observed in our data.
- $183\ 00:09:53.300 \dashrightarrow 00:09:57.200$  So our goal is to estimate the causal effect of A,
- 184 00:09:57.200 --> 00:10:00.610 treatment initiation time and T, death time.
- $185\ 00:10:00.610 \longrightarrow 00:10:04.590$  And we have almost 5,000 patients
- $186\ 00:10:04.590 \longrightarrow 00:10:07.160$  and only a very small proportion of patients
- $187\ 00:10:07.160 \longrightarrow 00:10:10.103$  have both observed A and T.
- 188 00:10:10.103 --> 00:10:12.930 A lot of patients don't have observed T.
- $189~00:10:12.930 \dashrightarrow 00:10:15.510$  So their death time is censored by C.
- $190\ 00:10:15.510 \longrightarrow 00:10:18.030$  And we have about 20% of our patients,
- 191 00:10:18.030 --> 00:10:20.256 they don't even have observed A.
- 192 00:10:20.256 --> 00:10:21.920 Their treatment initiation time
- $193\ 00:10:21.920 \longrightarrow 00:10:26.550$  can be censored by death time or censored by C, dropout,
- $194\ 00:10:26.550 \longrightarrow 00:10:27.513$  for example.
- 195 00:10:28.360 --> 00:10:32.160 So our goal is to estimate effect of A on T,
- 196 00:10:32.160 --> 00:10:34.530 but we only have about 300 patients
- $197\ 00:10:34.530 \longrightarrow 00:10:36.250$  have complete information.
- $198\ 00{:}10{:}36.250 {\:{\mbox{--}}}{>} 00{:}10{:}39.590$  Most of the patients we have incomplete information
- $199\ 00:10:39.590 \longrightarrow 00:10:42.500$  on either A or T or both.

- $200\ 00{:}10{:}42.500$  -->  $00{:}10{:}46.090$  How do we probably use these incomplete information
- 201 00:10:46.090 --> 00:10:49.360 to draw causal inference about A on T,
- $202\ 00:10:49.360 \longrightarrow 00:10:50.860$  the effect of A on T,
- $203\ 00:10:50.860 \longrightarrow 00:10:54.693$  that's a problem we solve in this project.
- $204\ 00:10:55.954 \longrightarrow 00:10:59.230$  So three challenges.
- 205 00:10:59.230 --> 00:11:02.160 First one, treatment initiation time,
- $206\ 00:11:02.160 --> 00:11:05.600$  this is observational data, so it's not randomly allocated.
- $207~00{:}11{:}05.600 \dashrightarrow 00{:}11{:}09.940$  We don't know the actual functional form of causal effect
- 208 00:11:09.940 --> 00:11:13.190 of initiation timing or mortality rate.
- $209\ 00{:}11{:}13.190 \dashrightarrow 00{:}11{:}16.890$  And we see that, Oh, there's incomplete information
- $210\ 00:11:16.890 \longrightarrow 00:11:20.073$  on either exposure or outcome or both.
- $211\ 00:11:21.500 \longrightarrow 00:11:23.680$  The general solutions we proposed
- $212\ 00:11:26.010 \longrightarrow 00:11:29.450$  that we first formulate a flexible structural
- 213 00:11:29.450 --> 00:11:30.990 causal hazard model
- $214\ 00:11:30.990 \longrightarrow 00:11:35.990$  that can capture the effects of both timing and duration
- $215\ 00:11:36.130 \longrightarrow 00:11:36.963$  of the treatment.
- $216\ 00:11:36.963 \longrightarrow 00:11:39.370$  And then we can derive methods
- $217\ 00:11:39.370 \longrightarrow 00:11:43.780$  to consistently estimate the model parameters
- $218\ 00{:}11{:}43.780 --> 00{:}11{:}48.290$  under non random allocation and complex censoring patterns.
- $219\ 00{:}11{:}48.290 \dashrightarrow 00{:}11{:}52.900$  Using the model outputs we can estimate the functional form
- $220\ 00{:}11{:}52.900 \dashrightarrow 00{:}11{:}56.452$  of the causal relationship between our initiation timing
- 221 00:11:56.452 --> 00:11:58.023 and mortality.
- $222\ 00{:}11{:}58.920 \dashrightarrow 00{:}12{:}02.463$  So some notation before we introduce our approach,
- $223\ 00{:}12{:}02.463 \dashrightarrow 00{:}12{:}06.300$  note that we have three time to events in our study,

- 224 00:12:06.300 --> 00:12:08.500 we have treatment initiation time, death time,
- 225 00:12:08.500 --> 00:12:09.690 censoring time.
- 226 00:12:09.690 --> 00:12:12.890 We'll use T sub cap A to denote death time
- $227\ 00:12:12.890 \longrightarrow 00:12:17.220$  associated with the actual treatment time.
- 228 00:12:17.220 --> 00:12:19.900 And potential outcomes T sub small A,
- $229\ 00:12:19.900 \longrightarrow 00:12:21.180$  this is the death time.
- 230 00:12:21.180 --> 00:12:24.360 If treatment initiated at time A,
- $231\ 00:12:24.360 \longrightarrow 00:12:27.600$  and we use T infinity to denote death time
- $232\ 00:12:27.600 \longrightarrow 00:12:30.950$  if treatment is initiated beyond sometime point
- $233\ 00:12:30.950 \longrightarrow 00:12:32.193$  of our interest.
- 234 00:12:33.269 --> 00:12:36.380 Because of all the censoring,
- $235\ 00:12:36.380 \longrightarrow 00:12:39.950$  all the three time to events can be censored by one another.
- 236 00:12:39.950 --> 00:12:44.100 We use T star to denote the minimum of T and C.
- 237 00:12:44.100 --> 00:12:47.060 Delta T is a corresponding event indicator.
- 238 00:12:47.060  $\rightarrow$  00:12:50.600 So A star is the minimum of the three time to events.
- $239\ 00:12:50.600 --> 00:12:53.838$  Delta A is a corresponding event in the data.
- $240\ 00{:}12{:}53.838 \dashrightarrow 00{:}12{:}57.600$  Adopting the convention in the causal inference literature,
- 241 00:12:57.600 --> 00:13:00.220 we use overbar to denote history.
- 242 00:13:00.220 --> 00:13:05.220 So overbar L of T here is a covariate history
- $243\ 00:13:05.880 \longrightarrow 00:13:07.960$  up to a time T.
- 244 00:13:07.960 --> 00:13:09.210 Putting everything together,
- $245\ 00:13:09.210 \longrightarrow 00:13:12.040$  we have a set of observed data.
- 246 00:13:12.040 --> 00:13:14.990 Now back to the censoring patterns.
- $247\ 00:13:14.990 \longrightarrow 00:13:18.210$  In case one, we observed both A and T.
- 248 00:13:18.210 --> 00:13:21.683 So we would observe A, we would observe T sub A.
- 249 00:13:22.630 --> 00:13:25.310 Case two T is censored by C,
- $250\ 00:13:25.310 --> 00:13:27.660$  so we observe A, we just know TA

- $251\ 00:13:27.660 \longrightarrow 00:13:29.390$  is going to be greater than C.
- 252 00:13:29.390 --> 00:13:30.502 Case three,
- $253\ 00:13:30.502 \longrightarrow 00:13:32.180$  we will observe A,
- $254\ 00:13:32.180 \longrightarrow 00:13:34.710$  but we know A is greater than TA.
- $255~00{:}13{:}34.710 \dashrightarrow 00{:}13{:}38.170$  And case four we don't observe A, we don't observe T
- $256~00{:}13{:}38.170 --> 00{:}13{:}41.723$  but we know A is greater than C and TA is greater than C.
- 257 00:13:42.850 --> 00:13:43.683 Okay.
- $258\ 00:13:43.683 --> 00:13:46.020$  So now we propose a structural causal
- $259\ 00:13:46.020 --> 00:13:47.840$  proportional hazards model
- $260\ 00:13:48.970 --> 00:13:52.493$  to capture the survival effect of treatment initiation time.
- 261 00:13:53.520 --> 00:13:55.690 Lambda AT here is a hazard function
- 262 00:13:55.690 --> 00:13:58.510 for the potential outcome T sub A,
- $263\ 00:13:58.510 \longrightarrow 00:14:01.320$  we start from lambda infinity T right here.
- 264 00:14:01.320 --> 00:14:04.514 This is a reference hazard for T infinity.
- $265\ 00:14:04.514 \longrightarrow 00:14:06.610$  So we start from here.
- $266\ 00:14:06.610 --> 00:14:10.040$  Once the treatment is initiated at A,
- $267\ 00{:}14{:}10.040 \dashrightarrow 00{:}14{:}13.910$  there is an instantaneous effect of treatment initiation
- $268\ 00:14:13.910 \longrightarrow 00:14:16.870$  captured by the G1 function here,
- 269 00:14:16.870 --> 00:14:19.990 and the effect of staying on the treatment
- $270\ 00:14:19.990 \longrightarrow 00:14:22.360$  at any given time point T,
- $271~00{:}14{:}22.360 \dashrightarrow 00{:}14{:}27.000$  is captured by the G2 function of ART duration.
- $272\ 00{:}14{:}27.000 \dashrightarrow 00{:}14{:}30.260$  And the G3 function here captures the interaction
- $273\ 00{:}14{:}30.260 \dashrightarrow 00{:}14{:}34.323$  between treatment initiation and treatment duration.
- $274\ 00:14:35.470 \longrightarrow 00:14:40.470$  So we leave this structural model relatively flexible.
- 275 00:14:40.790 --> 00:14:44.360 First, the reference hazard is left unspecified

 $276\ 00:14:44.360 \longrightarrow 00:14:46.570$  and the 3G functions, we also left them

277 00:14:46.570 --> 00:14:49.140 as unspecified smooth function

 $278\ 00:14:49.140 --> 00:14:53.233$  of treatment initiation time duration and their interaction.

 $279\ 00{:}14{:}54{.}220 \dashrightarrow 00{:}14{:}57.600$  So now we can parametrize these three functions

280 00:14:57.600 --> 00:14:59.754 using natural cubic splines,

 $281\ 00:14:59.754 \longrightarrow 00:15:04.754$  and by rewriting the risk function of our structural model,

 $282\ 00:15:06.640 \longrightarrow 00:15:09.460$  we can use beta this parameter

 $283\ 00:15:09.460 --> 00:15:13.140$  to include the causal effects of ART initiation time

 $284\ 00:15:13.140 \longrightarrow 00:15:14.870$  on mortality hazard.

285 00:15:14.870 --> 00:15:17.180 The problem here now,

286 00:15:17.180 --> 00:15:19.860 our goal is to how do we obtain a consistent

287 00:15:19.860 --> 00:15:22.653 estimate of beta using observed a data?

 $288\ 00:15:23.690 \longrightarrow 00:15:25.660$  Once we have obtained that

 $289\ 00{:}15{:}25.660 \dashrightarrow 00{:}15{:}29.990$  we can use beta hat to estimate the 3G functions,

290 00:15:29.990 --> 00:15:33.440 to understand the relative contribution of timing

 $291~00:15:33.440 \dashrightarrow 00:15:36.810$  versus duration and interactions.

 $292\ 00{:}15{:}36.810 \dashrightarrow 00{:}15{:}40.520$  And we could also estimate the causal does response

 $293\ 00:15:40.520 \longrightarrow 00:15:43.230$  of initiation time versus mortality

 $294\ 00:15:43.230 --> 00:15:46.450$  by relating the survival function to the hazard function.

 $295\ 00:15:46.450 \longrightarrow 00:15:51.283$  We can derive this from our structural model.

 $296\ 00:15:52.160 --> 00:15:54.640$  And now we can also estimate the model-based

 $297\ 00:15:54.640 \longrightarrow 00:15:56.650$  optimal initiation time

 $298~00{:}15{:}56.650 \dashrightarrow 00{:}16{:}01.330$  that will lead to the maximal survival probability

299 00:16:01.330 --> 00:16:05.423 at say 52 weeks after diagnosis.

- $300~00:16:06.500 \dashrightarrow 00:16:10.170$  Okay, how to obtain a consistent estimate of beta.
- 301 00:16:10.170 --> 00:16:14.950 So first let's assume if A is randomly allocated
- $302\ 00:16:14.950 \longrightarrow 00:16:17.410$  and both A and T are observed,
- $303\ 00{:}16{:}17.410 --> 00{:}16{:}21.910$  then we can write the partial likelihood score function
- $304\ 00:16:21.910 \longrightarrow 00:16:24.120$  of our structural model.
- $305\ 00:16:24.120 --> 00:16:28.135$  And this is a sample average of score function
- $306\ 00:16:28.135 \longrightarrow 00:16:31.550$  is an unbiased estimator of the expectation
- $307\ 00:16:31.550 \longrightarrow 00:16:32.730$  of the score function.
- $308\ 00:16:32.730 --> 00:16:36.508$  So E sub R here is the expectation
- $309\ 00:16:36.508 \longrightarrow 00:16:39.950$  under the randomized treatment assignment.
- $310\ 00:16:39.950 \longrightarrow 00:16:44.950$  So this would be an unbiased estimator function,
- $311\ 00:16:46.550 \longrightarrow 00:16:50.070$  and solving this unbiased estimating equation
- $312\ 00:16:50.070 \longrightarrow 00:16:53.153$  would give us a consistent estimator of beta.
- 313 00:16:54.760 --> 00:16:57.900 Now, if A is still randomly allocated,
- $314\ 00:16:57.900 \longrightarrow 00:17:00.173$  but T can occur before A,
- $315\ 00:17:01.933 \longrightarrow 00:17:03.823$  so A may be censored by T.
- 316 00:17:04.830 --> 00:17:05.670 In this case,
- $317\ 00:17:05.670 \longrightarrow 00:17:08.050$  we would need to break the mean
- $318\ 00{:}17{:}08.050 \dashrightarrow 00{:}17{:}11.280$  of an individual score contribution into two parts.
- $319\ 00:17:11.280 \longrightarrow 00:17:13.170$  In one part A is observed.
- $320\ 00:17:13.170 \longrightarrow 00:17:15.740$  The second part is A is not observed.
- $321\ 00{:}17{:}15.740 \dashrightarrow 00{:}17{:}18.890$  And then we can apply the law of total expectation
- $322\ 00:17:18.890 \longrightarrow 00:17:21.100$  to the second part.
- 323 00:17:21.100 --> 00:17:23.840 The inner expectation would be conditioning
- $324\ 00:17:23.840 \longrightarrow 00:17:26.390$  on the observed information.
- 325 00:17:26.390 --> 00:17:30.230 Then using this strategy and taking in account
- 326 00:17:30.230 --> 00:17:32.220 the survival hazard structure,

- $327\ 00:17:32.220 \longrightarrow 00:17:37.220$  we can revise the estimating equation.
- $328\ 00:17:37.350$  --> 00:17:41.253 And by solving this to obtain a consistent estimate of beta.
- $329\ 00:17:42.470 \longrightarrow 00:17:45.660$  In the case of non random allocation of treatment,
- 330 00:17:45.660 --> 00:17:50.380 then if we want to estimate the causal effect of A on T,
- $331\ 00:17:50.380 --> 00:17:53.943$  then we would have to make a key assumption,
- $332\ 00:17:55.600 \longrightarrow 00:17:57.150$  ignore ability assumption.
- $333\ 00:17:57.150 --> 00:17:58.620$  Essentially the assumption says
- 334 00:17:58.620 --> 00:18:03.620 that the initiation of treatment at any given time T
- $335\ 00:18:04.190 \longrightarrow 00:18:06.400$  is sequentially randomized in the sense
- $336\ 00:18:06.400 \longrightarrow 00:18:09.060$  that as a potential outcome beyond this time
- $337\ 00:18:09.060 --> 00:18:11.870$  is independent of treatment initiation.
- $338\ 00:18:11.870 \longrightarrow 00:18:15.930$  Conditioning on all covariate history up to T.
- $339\ 00:18:15.930 \longrightarrow 00:18:17.363$  So with this assumption,
- $340\ 00:18:18.610 --> 00:18:21.110$  we will be able to use observed data
- $341\ 00:18:21.110 \longrightarrow 00:18:23.180$  to derive the causal effect.
- $342\ 00:18:23.180$  --> 00:18:27.460 So say PR is the data distribution under randomized A,
- $343~00{:}18{:}27.460 \dashrightarrow 00{:}18{:}29.940$  and PO is the data distribution.
- $344\ 00:18:29.940 \longrightarrow 00:18:33.120$  And they're not random allocation of A.
- $345\ 00:18:33.120 \longrightarrow 00:18:35.440$  Note that in both settings,
- $346\ 00:18:35.440 \longrightarrow 00:18:38.710$  there is a same set of observed data.
- $347\ 00:18:38.710 --> 00:18:42.650$  And as long as the observed data under PR
- $348\ 00:18:42.650 --> 00:18:47.650$  is absolutely continues with the observed data under PO.
- $349\ 00{:}18{:}48.170 --> 00{:}18{:}51.940$  Now we can derive a random-nikodym derivative.
- 350 00:18:51.940 --> 00:18:54.500 And so Murphy's 2001 paper
- 351 00:18:54.500 --> 00:18:57.730 developed a version of R-N derivative

- $352\ 00:18:57.730 --> 00:19:01.360$  that connects the distribution of the observed data
- 353 00:19:01.360 --> 00:19:04.710 under PR and under PO for discrete time
- 354 00:19:04.710 --> 00:19:06.840 and ordinary GEE score.
- 355~00:19:06.840 --> 00:19:11.840 Johnson's 2005 paper extended this version of R-N derivative
- $356\ 00:19:11.950 --> 00:19:15.470$  to continuous time still for ordinary GEE score.
- $357\ 00:19:15.470 --> 00:19:20.287$  In this paper we extended the R-N derivative
- $358\ 00:19:20.287 \longrightarrow 00:19:23.451$  for time to event setting.
- $359\ 00:19:23.451 \longrightarrow 00:19:26.350$  So this is a version of R-N derivative
- $360\ 00:19:26.350 \longrightarrow 00:19:28.084$  for survival data.
- 361~00:19:28.084 --> 00:19:31.940 The reason why we wanted to use R-N derivative
- $362\ 00:19:31.940 \longrightarrow 00:19:34.080$  is that we can then use it
- 363 00:19:34.080 --> 00:19:36.840 to derive an unbiased estimating equation
- $364~00{:}19{:}36.840 \dashrightarrow 00{:}19{:}40.930$  using some weighted version of the observed data.
- $365\ 00:19:40.930 \longrightarrow 00:19:43.490$  So we can estimate the causal effect.
- 366~00:19:43.490 --> 00:19:48.490 So now we want to use this R-N derivative for survival data.
- $367\ 00:19:48.628 --> 00:19:51.380$  We want to apply that to Cox score
- $368\ 00:19:51.380 \longrightarrow 00:19:54.760$  and to derive S rated estimating equation.
- $369\ 00:19:54.760 \longrightarrow 00:19:59.550$  That's a little bit more complex than the GEE score,
- $370\ 00:19:59.550 --> 00:20:01.770$  but we can observe that the Cox score
- $371\ 00{:}20{:}01.770 \dashrightarrow 00{:}20{:}06.300$  can essentially be represented in three averages.
- $372\ 00:20:06.300 \longrightarrow 00:20:07.710$  The one in blue,
- $373\ 00:20:07.710 \longrightarrow 00:20:12.670$  the one in orange and the whole average.
- $374\ 00:20:12.670 \longrightarrow 00:20:17.240$  And each average converges to its expectation.
- $375\ 00:20:17.240 \longrightarrow 00:20:19.080$  And as I showed earlier,
- $376\ 00{:}20{:}19.080 \dashrightarrow 00{:}20{:}22.550$  we can always break the expectation into two parts.

- $377\ 00:20:22.550 \longrightarrow 00:20:24.770$  In one part A is observed,
- $378\ 00:20:24.770 \longrightarrow 00:20:26.880$  second part is not observed.
- $379\ 00:20:26.880 \longrightarrow 00:20:28.070$  For the second part,
- $380\ 00:20:28.070 \longrightarrow 00:20:32.418$  we can apply the total law of expectation,
- $381\ 00:20:32.418 \longrightarrow 00:20:34.700$  the law of total expectation,
- $382\ 00:20:34.700 \longrightarrow 00:20:39.700$  and recognizing the survival structure
- $383\ 00:20:40.310 \longrightarrow 00:20:42.560$  to derive the second part.
- $384~00{:}20{:}42.560 \dashrightarrow 00{:}20{:}46.340$  And then we can apply the R-N derivative for survival data
- 385 00:20:46.340 --> 00:20:48.400 to each piece separately,
- $386\ 00:20:48.400 \longrightarrow 00:20:52.143$  to construct the unbiased score equation.
- $387\ 00{:}20{:}53.390 \dashrightarrow 00{:}20{:}58.390$  So after some derivation, we would arrive at the weights
- $388\ 00{:}20{:}59.110 \dashrightarrow 00{:}21{:}03.480$  and actually the weights come down in a very neat form.
- 389 00:21:03.480 --> 00:21:06.040 Essentially, it suggests that for patients
- 390 00:21:06.040 --> 00:21:09.630 who have initiated treatment by time T,
- $391~00{:}21{:}09.630 \dashrightarrow 00{:}21{:}12.960$  we would weight them by the marginals density function
- $392\ 00:21:12.960 \longrightarrow 00:21:17.500$  of A divided by the conditional density of A
- 393 00:21:17.500 --> 00:21:22.500 given their covariate history after time T.
- 394 00:21:22.710 --> 00:21:25.080 And for those who are censored,
- $395\ 00:21:25.080 \longrightarrow 00:21:27.630$  so not initiated by the time T,
- $396~00{:}21{:}27.630 \dashrightarrow 00{:}21{:}30.280$  we would weight them by some survival function
- $397\ 00:21:31.229 --> 00:21:34.673$  of the treatment initiation process.
- $398\ 00:21:36.120 \longrightarrow 00:21:38.910$  And then by applying this weighting scheme,
- $399\ 00{:}21{:}38.910 \dashrightarrow 00{:}21{:}43.300$  we will be able to derive a weighted estimating equation.
- $400\ 00:21:43.300 \longrightarrow 00:21:45.880$  And just a note that we have to apply
- $401\ 00:21:45.880 \longrightarrow 00:21:49.370$  the same weighting scheme to the people
- $402\ 00:21:49.370 \longrightarrow 00:21:52.653$  who are still in the risk set at any time T.

- $403\ 00{:}21{:}54.310 {\:{\mbox{--}}\!>\:} 00{:}21{:}57.750$  And so now that said, previously we have assumed
- 404 00:21:57.750 --> 00:21:58.900 there's no censoring.
- 405 00:21:58.900 --> 00:22:00.368 Now with censoring,
- $406\ 00:22:00.368 --> 00:22:05.368$  we need to assume another similar assumption,
- 407 00:22:05.895 --> 00:22:08.824 similar to the ignore ability assumption,
- 408 00:22:08.824 --> 00:22:12.140 and then using the similar strategy
- $409\ 00:22:12.140 \longrightarrow 00:22:15.013$  to derive another set of weight for censoring.
- $410\ 00:22:16.050 \longrightarrow 00:22:18.450$  For those who stay, remain in the study,
- $411\ 00:22:18.450 --> 00:22:22.520$  we would weight them by the survival function
- 412 00:22:22.520 --> 00:22:23.940 for censoring.
- 413 00:22:23.940 --> 00:22:26.190 And this would lead to the final modification
- $414\ 00:22:27.136 \longrightarrow 00:22:29.470$  of the estimating equation for beta.
- $415\ 00{:}22{:}29.470 --> 00{:}22{:}33.330$  So censoring contributes information about the parameter
- $416\ 00:22:33.330 \longrightarrow 00:22:34.770$  in two ways,
- $417\ 00{:}22{:}34.770 \dashrightarrow 00{:}22{:}39.510$  FC is observed as the person is actually censored.
- $418\ 00:22:39.510 \longrightarrow 00:22:42.250$  It contributes to the risk set up to C.
- 419 00:22:42.250 --> 00:22:45.960 If C is not observed, so C could be censored by T.
- 420 00:22:45.960 --> 00:22:47.210 If death's occurred,
- $421\ 00:22:47.210 \longrightarrow 00:22:49.990$  then it contributes to the individual partial likelihood
- $422\ 00:22:49.990 \longrightarrow 00:22:53.633$  to weight for C but evaluated at death time.
- 423 00:22:54.810 --> 00:22:56.460 Okay, now we know how to weight.
- $424~00:22:56.460 \longrightarrow 00:22:58.640$  Back to the four censoring patterns.
- 425 00:22:58.640 --> 00:23:01.530 The first one, both A and T are observed.
- 426 00:23:01.530 --> 00:23:06.110 We would weight them by the first set of weight for A
- 427 00:23:06.110 --> 00:23:06.943 evaluated at A,
- $428~00{:}23{:}08.170 \dashrightarrow 00{:}23{:}13.170$  T occurred, so the weight for C but evaluated at T.

- 429 00:23:13.360 --> 00:23:16.610 Second case, T is not observed,
- $430\ 00:23:16.610 \longrightarrow 00:23:19.100$  A is observed.
- 431 00:23:19.100 --> 00:23:23.340 So first set of weight for A evaluated at A
- $432\ 00{:}23{:}23.340 \dashrightarrow 00{:}23{:}26.163$  and C just contributes information to the risks set.
- 433 00:23:28.260 --> 00:23:31.370 Third case, A is not observed,
- $434\ 00:23:31.370 \longrightarrow 00:23:35.220$  so second weight for A evaluated at T.
- 435 00:23:35.220 --> 00:23:40.220 And weight for C, censoring evaluated at T.
- $436\ 00:23:40.250 \longrightarrow 00:23:43.820$  The fourth case or final case, A is not observed,
- 437 00:23:43.820 --> 00:23:46.780 again, second set of weight for A,
- $438\ 00:23:46.780 --> 00:23:50.453$  but evaluated at C, and C also contributes to the risks set.
- 439 00:23:51.420 --> 00:23:53.930 Okay, so now we know how to weight.
- $440\ 00:23:53.930 --> 00:23:57.843$  We would have to estimate the weights.
- 441 00:24:00.320 --> 00:24:02.490 The approach we used in the paper
- $442\ 00:24:02.490 \longrightarrow 00:24:05.540$  is that we model the intensity processes
- 443 00:24:05.540 --> 00:24:09.410 associated with the two counting processes,
- $444\ 00:24:09.410 \longrightarrow 00:24:11.500$  one for A, one for C.
- $445\ 00{:}24{:}11.500 \dashrightarrow 00{:}24{:}14.680$  And then when we fit Cox proportional hazards models
- 446 00:24:14.680 --> 00:24:17.220 for the two intensity processes,
- $447\ 00:24:17.220 \longrightarrow 00:24:20.053$  we use fitted hazard to estimate the weights.
- 448 00:24:21.110 --> 00:24:23.680 We use empirical cumulative hazards
- $449\ 00:24:23.680 \longrightarrow 00:24:26.810$  to estimate the conditional density and function.
- 450 00:24:26.810 --> 00:24:28.760 And for the marginal density function,
- $451\ 00{:}24{:}28.760 \dashrightarrow 00{:}24{:}32.110$  we use some nonparametric Nelson-Aalen estimator,
- $452\ 00:24:32.110 \longrightarrow 00:24:34.910$  and use similar fashion to estimate rates for censoring.
- $453\ 00{:}24{:}36.220 \operatorname{--}{>} 00{:}24{:}39.380$  Then we apply our methods to the AMPATH data.

- $454\ 00{:}24{:}39.380 \dashrightarrow 00{:}24{:}44.210$  AMPATH is a large HIV care program based in West Kenya,
- 455 00:24:44.210 --> 00:24:47.100 our data has almost 5,000 patients
- $456\ 00{:}24{:}47.100 \dashrightarrow 00{:}24{:}51.017$  and for covariates, we have demographic information
- $457\ 00:24:51.017 \longrightarrow 00:24:53.590$  and some disease-specific information.
- $458\ 00:24:53.590 --> 00:24:56.440$  Some of them are time varying like, weight, the CD4,
- $459\ 00:24:56.440 \longrightarrow 00:24:58.890$  these are time varying variables.
- $460\ 00{:}24{:}58.890 \dashrightarrow 00{:}25{:}03.777$  We categorize the baseline CD4 subgroups into two groups,
- $461\ 00:25:05.980 \longrightarrow 00:25:08.310$  the less than, or below 50 group,
- $462\ 00:25:08.310 \longrightarrow 00:25:10.900$  this is the highest risk group.
- $463\ 00:25:10.900 \longrightarrow 00:25:13.600$  So CD4 the higher, the better.
- 464 00:25:13.600 --> 00:25:16.170 So below 50, this is a highest risk group.
- 465 00:25:16.170 --> 00:25:18.690 And between 200 and 350,
- 466 00:25:18.690 --> 00:25:20.890 there's relatively healthy patients.
- $467\ 00{:}25{:}20.890 {\: \hbox{--}}{>}\ 00{:}25{:}23.200$  The reason we categorize them into three groups
- $468\ 00:25:23.200 \longrightarrow 00:25:26.190$  is because the program guidelines
- $469\ 00:25:26.190 \longrightarrow 00:25:28.180$  are based on these subgroups
- $470\ 00{:}25{:}28.180$  -->  $00{:}25{:}31.733$  and RCT is reported results for below 50 group.
- $471\ 00{:}25{:}33.039 \dashrightarrow 00{:}25{:}37.030$  We want to compare our results to our CT findings.
- $472\ 00{:}25{:}37.030 \dashrightarrow 00{:}25{:}41.950$  So this plot shows the three estimated G functions.
- $473\ 00:25:41.950 --> 00:25:46.830$  The G1 A here suggests that the instantaneous effect
- 474 00:25:46.830 --> 00:25:49.920 of a treatment initiation has a U shape,
- $475\ 00{:}25{:}49.920 \dashrightarrow 00{:}25{:}53.290$  achieving maximum benefit, or the lowest mortality hazard
- $476\ 00:25:53.290 \longrightarrow 00:25:55.620$  at just about 10 weeks.

- $477\ 00:25:55.620 \longrightarrow 00:25:59.630$  And after that, the longer the treatment is delayed,
- $478\ 00:25:59.630 \longrightarrow 00:26:03.170$  the less the benefit of the treatment initiation.
- $479\ 00:26:03.170 \longrightarrow 00:26:05.510$  And this is the effect of duration,
- 480 00:26:05.510 --> 00:26:07.660 in general, it says that the longer
- $481\ 00:26:07.660 \longrightarrow 00:26:10.700$  you stay on the treatment, the more benefit you get.
- $482\ 00{:}26{:}10.700 \dashrightarrow 00{:}26{:}15.170$  There's an upward trend for the interaction effect.
- $483\ 00{:}26{:}15.170 \dashrightarrow 00{:}26{:}18.830$  Essentially suggesting that delayed treatment initiation
- 484 00:26:18.830 --> 00:26:21.800 would reduce the benefit associated
- 485~00:26:21.800 --> 00:26:25.023 with long ART duration.
- $486\ 00{:}26{:}26{.}990 \dashrightarrow 00{:}26{:}30{.}930$  And so the net causal effect of treatment initiation
- $487\ 00:26:30.930 \longrightarrow 00:26:33.310$  is summarized in this plot.
- $488\ 00:26:33.310 \longrightarrow 00:26:37.570$  Top panel shows the mortality rate at one year
- $489\ 00:26:37.570 --> 00:26:40.210$  versus treatment initiation time.
- $490\ 00:26:40.210 \longrightarrow 00:26:44.320$  Bottom panel compares immediate initiation
- $491\ 00:26:44.320 \longrightarrow 00:26:47.990$  versus delayed initiation at A.
- $492\ 00{:}26{:}47.990 \to 00{:}26{:}52.670$  So we can see that the benefit of early initiation
- $493\ 00:26:52.670 --> 00:26:56.650$  is most pronounced for the CD4 below 50 group,
- $494\ 00:26:56.650 \longrightarrow 00:26:58.310$  or the highest risk group.
- $495\ 00:26:58.310 \longrightarrow 00:27:00.550$  And the curves here are pretty flat,
- $496~00:27:00.550 \longrightarrow 00:27:03.077$  suggesting that there's not much benefit
- $497\ 00{:}27{:}03.077 \dashrightarrow 00{:}27{:}06.773$  of early initiation for relatively healthy patients.
- $498\ 00:27:08.770 --> 00:27:12.063$  Several advantages for this approach.
- 499 00:27:12.063 --> 00:27:16.960 It's easy to get optimal initiation time
- $500\ 00:27:16.960 \longrightarrow 00:27:19.123$  based on the model outputs.
- $501\ 00:27:20.325 --> 00:27:22.410$  And we could also use the model outputs

- $502\ 00:27:22.410$  --> 00:27:27.370 to emulate comparisons between regimens reported in RCTs.
- $503\ 00:27:27.370 --> 00:27:31.940$  So we could mimic random allocation
- $504~00{:}27{:}31.940 \dashrightarrow 00{:}27{:}35.690$  of treatment initiation time to specific intervals
- 505 00:27:35.690 --> 00:27:38.670 by assuming a distribution for A,
- 506 00:27:38.670 --> 00:27:41.170 for treatment initiation time A,
- $507\ 00:27:41.170 --> 00:27:44.020$  that is independent of covariates and outcome
- 508 00:27:44.020 --> 00:27:48.820 and compare interval specific mortality rates
- $509~00{:}27{:}48.820 --> 00{:}27{:}53.180$  and draw inferences about treatment initiation.
- $510~00{:}27{:}53.180 \dashrightarrow 00{:}27{:}56.210$  But with the continuous time marginal structure model,
- $511\ 00:27:56.210 --> 00:28:00.070$  we'll also be able to conduct a higher resolution analysis
- 512 00:28:00.070 --> 00:28:02.620 that can potentially generate new insights
- $513\ 00:28:02.620 \longrightarrow 00:28:05.893$  in relation to a randomized control trial.
- 514 00:28:09.160 --> 00:28:10.480 For the sake of timing,
- $515\ 00:28:10.480 \longrightarrow 00:28:13.630$  I just gonna briefly talk about the simulation.
- 516 00:28:13.630 --> 00:28:15.440 We conduct simulation to examine
- $517~00{:}28{:}15.440 \dashrightarrow 00{:}28{:}18.800$  the finite-sample properties of weighted estimators,
- $518\ 00:28:23.890 \longrightarrow 00:28:26.759$  we evaluate sensitivity of our estimators
- 519 00:28:26.759 --> 00:28:29.580 to the violations of the ignore ability,
- 520 00:28:29.580 --> 00:28:31.870 or no unmeasured confounding assumption,
- $521~00{:}28{:}31.870 \dashrightarrow 00{:}28{:}34.780$  but we only considered confounding at baseline.
- 522 00:28:34.780 --> 00:28:38.640 So the sensitivity analysis strategy
- 523 00:28:38.640 --> 00:28:41.120 for time-varying confounding,
- 524 00:28:41.120 --> 00:28:43.760 especially with the censored survival outcome
- 525 00:28:43.760 --> 00:28:48.220 is kind of very complex topic,
- $526\ 00:28:48.220$  --> 00:28:51.340 and we were still working on this project right now,

- $527\ 00:28:51.340 --> 00:28:54.683$  but in this paper we just consider confounding at baseline.
- 528 00:28:56.330 --> 00:28:58.690 Under random allocation of treatment,
- $529\ 00:28:58.690 --> 00:29:02.060$  our estimator produced a new zero bias
- 530 00:29:02.060 --> 00:29:04.910 and nominal coverage probability,
- 531 00:29:04.910 --> 00:29:06.990 in the presence of measured confounding,
- $532\ 00:29:06.990 \longrightarrow 00:29:09.260$  it eliminated nearly all the biases
- $533\ 00:29:09.260 \longrightarrow 00:29:12.949$  and provided close to nominal coverage probability,
- $534\ 00:29:12.949 \longrightarrow 00:29:16.130$  but in the presence of unmeasured confounding,
- $535\ 00:29:16.130 \longrightarrow 00:29:19.100$  there was bias in our estimator.
- $536\ 00:29:19.100 \longrightarrow 00:29:22.140$  And the biases were in proportion
- $537\ 00:29:22.140 --> 00:29:24.333$  to the degree of measured confounding.
- 538 00:29:26.490 --> 00:29:27.323 Okay,
- 539 00:29:27.323 --> 00:29:29.440 so moving to the second example,
- $540~00{:}29{:}29.440 \dashrightarrow 00{:}29{:}33.610$  this is a continuous time dynamic treatment regimen
- $541\ 00:29:33.610 \longrightarrow 00:29:34.443$  of the form,
- $542\ 00{:}29{:}34.443 \dashrightarrow 00{:}29{:}38.373$  initiate treatment when a biomarker crosses a threshold.
- $543\ 00:29:39.512 --> 00:29:41.930$  It's dynamic treatment regimen
- $544\ 00{:}29{:}41.930 {\: -->\:} 00{:}29{:}44.970$  because it depends on evolving history of treatment
- 545 00:29:44.970 --> 00:29:46.980 and a tailoring variable.
- $546\ 00:29:46.980 \longrightarrow 00:29:49.600$  So in our case, CD4 is a tailoring variable.
- $547\ 00:29:49.600 \longrightarrow 00:29:52.790$  That means we make our treatment decision
- $548\ 00:29:52.790 \longrightarrow 00:29:54.073$  based on this variable.
- $549~00{:}29{:}55.150 \dashrightarrow 00{:}30{:}00.100$  A little bit different from our previous motivating example.
- $550\ 00:30:00.100 \longrightarrow 00:30:03.580$  The outcome interest is different.
- $551\ 00:30:03.580 \longrightarrow 00:30:05.150$  This is a pediatric data.
- $552\ 00:30:05.150 \longrightarrow 00:30:08.980$  So for the kids, the mortality rate is very low

- $553\ 00:30:08.980 \longrightarrow 00:30:12.242$  and our data I think it's around 3%.
- 554 00:30:12.242 --> 00:30:14.470 And for kids, we're also interested
- 555 00:30:14.470 --> 00:30:17.220 in their CD4 measurements,
- 556~00:30:17.220 --> 00:30:21.300 because CD4 is important marker of immune system function
- 557~00:30:21.300 --> 00:30:24.260 and both outcomes, both mortality rate and  $\mathrm{CD4}$
- 558 00:30:24.260 --> 00:30:26.333 are sparsely measured in our data,
- $559\ 00:30:27.200 \longrightarrow 00:30:28.700$  but we are interested in both.
- $560\ 00:30:29.620 --> 00:30:32.790$  Other than that, we also have complications
- $561\ 00:30:32.790 \longrightarrow 00:30:36.250$  posed by observational data.
- $562\ 00:30:36.250 --> 00:30:41.250$  So this is a picture of nine randomly selected individuals
- 563 00:30:41.430 --> 00:30:42.670 from our data,
- 564 00:30:42.670 --> 00:30:45.900 X axis here, follow-up time in days,
- 565 00:30:45.900 --> 00:30:49.440 Y axis here square root of CD4,
- $566\ 00:30:49.440 --> 00:30:53.220$  purple line is end of follow-up,
- $567~00{:}30{:}53.220 \dashrightarrow 00{:}30{:}56.950$  two gray lines here mark one year
- $568\ 00:30:56.950 \longrightarrow 00:30:59.023$  and two years post diagnosis.
- $569\ 00:31:00.270 --> 00:31:03.950$  Empty circles here mean that the patient
- $570\ 00:31:03.950 \longrightarrow 00:31:06.010$  has not been treated.
- $571\ 00:31:06.010$  --> 00:31:09.310 Solid circles, mean that they're on the treatment.
- $572\ 00:31:09.310 --> 00:31:11.920$  So we can see that there's a lot of variability
- $573\ 00:31:11.920 \longrightarrow 00:31:15.940$  in terms of the treatment initiation time.
- $574\ 00:31:15.940 --> 00:31:19.620$  And some people are followed much longer
- 575~00:31:19.620 --> 00:31:22.290 than some other patients.
- $576~00{:}31{:}22.290 \dashrightarrow 00{:}31{:}27.290$  And the follow-up time is pretty irregularly spaced
- $577\ 00:31:29.370 \longrightarrow 00:31:33.880$  and overall the CD4 measurements are quite sparse,
- $578\ 00:31:33.880 --> 00:31:36.440$  and there's also incomplete information
- $579\ 00:31:36.440 \longrightarrow 00:31:41.440$  for example, these two they either died

- $580\ 00:31:41.490 --> 00:31:43.830$  or were lost to follow up
- $581\ 00:31:43.830 --> 00:31:47.410$  before they even got a chance to be treated.
- $582\ 00:31:47.410$  --> 00:31:51.382 So there's also a lot of complication in the data.
- 583 00:31:51.382 --> 00:31:53.640 There's a continuous time measurement
- $584\ 00:31:53.640 --> 00:31:55.380$  of the treatment initiation.
- $585\ 00:31:55.380 \longrightarrow 00:31:57.725$  It just happens all over the place.
- $586\ 00:31:57.725 \longrightarrow 00:32:02.600$  The longitudinal outcome of interest are sparsely measured,
- $587\ 00:32:02.600 --> 00:32:04.710$  leading to incomplete data.
- 588~00:32:04.710 --> 00:32:08.570 There's also a censoring due to dropout or deaths.
- $589~00:32:08.570 \dashrightarrow 00:32:11.410$  So our general solution is that we'll use weighting
- 590 00:32:11.410 --> 00:32:13.800 to handle time-varying confounding.
- $591\ 00:32:13.800$  --> 00:32:16.830 And will show how to derive a continuous time versions
- $592\ 00:32:16.830 \longrightarrow 00:32:18.820$  of the weights.
- $593\ 00:32:18.820 \longrightarrow 00:32:21.400$  For the missing outcomes
- $594\ 00{:}32{:}21.400 \dashrightarrow 00{:}32{:}24.470$  that is caused by sparse measurement and censoring
- 595~00:32:24.470 --> 00:32:27.920 we'll use imputations from a model of the joint distribution
- 596 00:32:27.920 --> 00:32:30.150 of CD4 and mortality.
- $597\ 00:32:30.150 --> 00:32:33.200$  And because we're interested in both mortality status
- 598 00:32:33.200 --> 00:32:36.363 and CD4, we'll develop a composite outcome.
- $599~00{:}32{:}37.970 \dashrightarrow 00{:}32{:}42.460$  So our general approach is to emulate a randomized trial
- $600\ 00:32:42.460 --> 00:32:45.000$  in which we would randomize individuals
- $601\ 00:32:45.000 \longrightarrow 00:32:47.900$  to follow specific DTR Q.
- 602 00:32:47.900 --> 00:32:50.950 And Q equals zero means never treated,
- $603\ 00:32:50.950 \longrightarrow 00:32:54.170$  because CD4 can never drop below zero.

- $604~00{:}32{:}54.170 \dashrightarrow 00{:}32{:}57.730$  Now, Q equals infinity means treat immediately.
- $605\ 00:32:57.730 \longrightarrow 00:32:59.340$  So after randomization,
- $606\ 00:32:59.340 \longrightarrow 00:33:02.460$  all the individuals will be followed
- 607 00:33:02.460 --> 00:33:04.890 for a fixed amount of time,
- 608 00:33:04.890 --> 00:33:07.160 at which point, say T star,
- $609\ 00:33:07.160 \longrightarrow 00:33:09.830$  both their mortality status.
- 610 00:33:09.830 --> 00:33:14.110 And among those who are alive at T star,
- $611\ 00:33:14.110 \longrightarrow 00:33:18.230$  their CD4 count will be assessed.
- $612\ 00:33:18.230 \longrightarrow 00:33:21.240$  So what define a composite outcome XQ,
- $613\ 00:33:21.240 \longrightarrow 00:33:25.120$  that is the product of the test indicator
- $614\ 00:33:25.120 \longrightarrow 00:33:26.643$  and the potential CD4.
- $615\ 00{:}33{:}27.770 \dashrightarrow 00{:}33{:}31.600$  So the cumulative distribution of this composite outcome
- 616 00:33:31.600 --> 00:33:35.610 is a useful measure of treatment utility,
- $617\ 00:33:35.610 \longrightarrow 00:33:38.580$  because it has appointments at zero
- 618 00:33:38.580 --> 00:33:40.850 corresponding to mortality rate.
- 619 00:33:40.850 --> 00:33:45.310 Thereby capturing both mortality status
- $620\ 00:33:45.310 --> 00:33:50.310$  and CD4 count among survivors at T star.
- $621~00{:}33{:}51.610 \dashrightarrow 00{:}33{:}55.720$  So for example, the probability of a positive XQ,
- 622 00:33:55.720 --> 00:33:57.980 that's the survival fraction,
- $623\ 00:33:57.980 \longrightarrow 00:34:01.510$  and the probability of XQ greater than X,
- $624\,00:34:01.510$  --> 00:34:06.053 that's the fraction of survivors with CD4 above X.
- $625~00{:}34{:}07.560 \dashrightarrow 00{:}34{:}11.960$  Okay, so similar to the first motivating example,
- $626\ 00:34:11.960 \longrightarrow 00:34:14.653$  we again have three timed events.
- $627~00{:}34{:}15.650 \dashrightarrow 00{:}34{:}19.170$  Death time, censoring time, treatment initiation time.
- $628~00{:}34{:}19.170 \dashrightarrow 00{:}34{:}22.590$  And now we have a tailoring variable, CD4 count.

- $629\ 00:34:22.590 \longrightarrow 00:34:26.930$  So the CD four process is defined for all continuous time,
- $630\ 00:34:26.930 \longrightarrow 00:34:30.310$  but it's just measured at discrete times.
- $631\ 00:34:30.310 \longrightarrow 00:34:33.863$  And we also have a P by one covariate process.
- 632 00:34:34.940 --> 00:34:37.760 Using a convention in the DTR literature,
- $633\ 00:34:37.760 \longrightarrow 00:34:40.288$  we assume that the treatment decision
- $634\ 00{:}34{:}40.288 {\: -->\:} 00{:}34{:}44.980$  is always made after observing the covariate history
- $635\ 00:34:44.980 \longrightarrow 00:34:48.419$  and the CD4 count.
- 636 00:34:48.419 --> 00:34:50.140 Putting everything together,
- $637\ 00:34:50.140 \longrightarrow 00:34:55.140$  we have a history information indicator.
- $638\ 00:34:55.140 --> 00:34:59.510$  For each individual, we'll have a observed a data process.
- $639\ 00:34:59.510 \longrightarrow 00:35:01.700$  And just note that each person
- $640\ 00:35:01.700 --> 00:35:03.770$  can have a different lens of followup
- $641\ 00:35:03.770 \longrightarrow 00:35:05.173$  at different time points.
- $642\ 00:35:08.100 --> 00:35:11.800$  Our goal is to evaluate the effect of DTRs,
- 643 00:35:11.800 --> 00:35:14.410 but we're dealing with observational data,
- $644~00:35:14.410 \longrightarrow 00:35:17.630$  so we'll have to map the observed treatment regimen
- $645\ 00{:}35{:}17.630 \dashrightarrow 00{:}35{:}21.927$  to specific DTRs that we are interested in evaluating.
- $646~00{:}35{:}21.927 \dashrightarrow 00{:}35{:}26.927$  Essentially we'll follow the deterministic function
- $647\ 00:35:28.090 \longrightarrow 00:35:29.570$  to create the mapping.
- $648\ 00:35:29.570 \longrightarrow 00:35:31.686$  Essentially there are three rules.
- $649\ 00:35:31.686 \longrightarrow 00:35:34.416$  First rule says not to treat the person
- $650\ 00:35:34.416 \longrightarrow 00:35:37.750$  if the person has not yet initiated treatment
- $651\ 00:35:37.750 \longrightarrow 00:35:40.870$  and their CD4 has not fallen below Q,
- 652 00:35:40.870 --> 00:35:42.403 or has not been observed.
- 653 00:35:43.710 --> 00:35:47.080 Second rule says, treat this person if their time T,
- $654\ 00:35:47.080 \longrightarrow 00:35:51.170\ \mathrm{CD4}$  has fallen below Q for the very first time.

- $655\ 00:35:51.170 \longrightarrow 00:35:53.920$  Once treated, always treat them.
- $656\ 00:35:53.920 \longrightarrow 00:35:55.510$  Following these three rules,
- $657\ 00:35:55.510 --> 00:35:59.880$  we'll be able to create a regimen specific compliant process
- $658\ 00:35:59.880 \longrightarrow 00:36:01.890$  for each individual in the data.
- $659\ 00:36:01.890 \longrightarrow 00:36:05.010$  So essentially if the rule says treat,
- $660\ 00:36:05.010$  --> 00:36:08.640 and if the person is actually treated by the time T,
- $661\ 00:36:08.640 --> 00:36:12.520$  then this person is compliant at time T.
- 662 00:36:12.520 --> 00:36:14.260 If the rule says do not treat,
- $663~00:36:14.260 \dashrightarrow 00:36:16.610$  and the person was not treated at the time T,
- $664\ 00:36:16.610 \longrightarrow 00:36:18.963$  so this person is still compliant to the rule.
- $665~00{:}36{:}20.090 \dashrightarrow 00{:}36{:}23.590$  And so we'll be able to observe a compliant process
- $666\ 00:36:23.590 \longrightarrow 00:36:25.101$  for each person.
- $667\ 00:36:25.101$  --> 00:36:29.430 Here a simple example to show you how to create the mapping.
- $668\ 00:36:29.430 \longrightarrow 00:36:33.307$  For example, we're interested in Q equals 350.
- $669\ 00:36:33.307 \longrightarrow 00:36:35.440$  This person came in at baseline,
- $670\ 00:36:35.440 \longrightarrow 00:36:38.620$  had a measurement 400 above the threshold.
- 671 00:36:38.620 --> 00:36:40.000 The rule says do not treat,
- $672\ 00:36:40.000 \longrightarrow 00:36:41.450$  the person was not treated.
- 673 00:36:41.450 --> 00:36:44.010 At this point, it's compliant with the rule.
- 674 00:36:44.010 --> 00:36:48.090 Next visit, no new CD4 observation.
- 675 00:36:48.090 --> 00:36:49.590 So the rule says do not treat,
- $676\ 00:36:49.590 \longrightarrow 00:36:51.695$  the person's still not treated,
- $677\ 00:36:51.695 \longrightarrow 00:36:53.030$  still compliant at this point.
- 678 00:36:53.030 --> 00:36:57.640 Third visit, the person's CD4 drops to 330,
- $679\ 00:36:57.640 \longrightarrow 00:37:01.490$  which is below the threshold for the very first time,
- 680 00:37:01.490 --> 00:37:04.540 the rules are start treating this person,
- $681\ 00:37:04.540 --> 00:37:06.370$  the person was actually treated.

- $682\ 00:37:06.370 \longrightarrow 00:37:09.610$  So compliant at this point.
- $683\ 00:37:09.610$  --> 00:37:14.370 Next visit the rule says once treated always treat them,
- $684\ 00:37:14.370 \longrightarrow 00:37:16.390$  the person kept being treated.
- $685~00{:}37{:}16.390 \dashrightarrow 00{:}37{:}19.820$  So this person was compliant with the rule 350
- $686\ 00:37:19.820 \longrightarrow 00:37:22.453$  all throughout his or her followup.
- $687\ 00:37:23.350 \longrightarrow 00:37:27.410$  Next example, the first two rows are the same.
- 688 00:37:27.410 --> 00:37:32.410 The third visit, the person's CD4 jumps to 450,
- $689\ 00:37:32.900 \longrightarrow 00:37:34.990$  which is above the threshold.
- $690\ 00:37:34.990 \longrightarrow 00:37:36.520$  The rule says do not treat,
- $691~00{:}37{:}36.520 \dashrightarrow 00{:}37{:}40.450$  but on the contrary, the person was actually treated
- $692\ 00:37:40.450 \longrightarrow 00:37:42.480$  and kept being treated.
- 693 00:37:42.480 --> 00:37:45.760 So from this time point onward,
- $694\ 00:37:45.760 --> 00:37:49.083$  the person was not compliant with this rule.
- $695\ 00:37:50.970 \longrightarrow 00:37:54.553$  Okay, so that's just some simple example
- $696\ 00:37:54.553 \longrightarrow 00:37:58.480$  to show how to create the mapping.
- 697 00:37:58.480 --> 00:38:01.240 With missing outcomes for those alive
- $698\ 00:38:01.240 --> 00:38:04.660$  at the target measurement time T star,
- 699 00:38:04.660 --> 00:38:09.660 the observed outcome XI is the CD4 measurement at T star.
- $700\ 00:38:10.800 --> 00:38:14.000$  But because of CD4 is sparsely measured
- 701 00:38:14.000 --> 00:38:16.540 and irregularly spaced,
- 702 00:38:16.540 --> 00:38:18.856 Z of T star is directly observed
- $703\,00:38:18.856$  --> 00:38:23.856 only when the person's follow up time is exactly at T star.
- 704 00:38:24.810 --> 00:38:27.250 So in this case, it is pretty common
- 705~00:38:27.250 --> 00:38:32.250 to predefine a interval and capture the CD4 that is measured
- 706 00:38:37.080 --> 00:38:40.920 at the time closest to the target measurement time.

- 707 00:38:40.920 --> 00:38:42.880 But even using this strategy,
- 708~00:38:42.880 --> 00:38:47.880 there's still a possibility that there is no measurement
- $709\ 00:38:48.374 \longrightarrow 00:38:51.310$  in predefined interval.
- 710 00:38:51.310 --> 00:38:53.990 Then we say this person has a missing outcome.
- 711 00:38:53.990 --> 00:38:56.690 And it's also possible that the person dropped out
- 712  $00:38:56.690 \longrightarrow 00:38:57.523$  before TA.
- 713 00:38:58.970 --> 00:39:02.483 And so in this case, the outcome is also missing.
- $714\ 00:39:03.815 --> 00:39:07.940$  For these missing outcomes, our general strategy
- $715\ 00:39:07.940 \longrightarrow 00:39:10.120$  is to use multiple imputation.
- $716\ 00:39:10.120 --> 00:39:12.280$  So we would specify and fit model
- 717 00:39:12.280 --> 00:39:15.630 for the joint distribution of the CD4 process
- $718\ 00:39:15.630 \longrightarrow 00:39:17.720$  and the mortality process.
- $719\ 00:39:17.720 \longrightarrow 00:39:20.300$  For those known to be alive,
- 720 00:39:20.300 --> 00:39:22.590 but without a CD4 measurement,
- 721 00:39:22.590 --> 00:39:27.590 we would impute the CD4 count from the fitted CD4 sub-model.
- $722\ 00:39:28.460 \longrightarrow 00:39:30.840$  And for those missing the CD4,
- 723 00:39:30.840 --> 00:39:32.820 because of right censoring,
- $724\ 00:39:32.820 \longrightarrow 00:39:37.110$  we would calculate the mortality probability
- 725 00:39:37.110 --> 00:39:38.890 from the fitted survival sub-model,
- $726\ 00:39:38.890 --> 00:39:41.310$  and then impute the death indicator
- 727 00:39:42.350 --> 00:39:43.750 from the Bernoulli distribution
- $728\ 00:39:43.750 --> 00:39:46.020$  with this calculated probability.
- $729\ 00:39:46.020 --> 00:39:48.830$  If the death indicator was imputed to be zero,
- $730\ 00:39:48.830 \longrightarrow 00:39:52.200$  then we further impute a CD4 count for this person.
- 731 00:39:52.200 --> 00:39:54.853 Otherwise we'll set X to be zero.
- $732\ 00:39:56.220 --> 00:39:58.950$  And again, we would have to assume

 $733\ 00:39:58.950 \longrightarrow 00:40:01.850$  some standard causal inference assumptions

734 00:40:01.850 --> 00:40:06.050 in order to draw causal effects about the DTRO

 $735\ 00:40:07.570 \longrightarrow 00:40:09.410$  using observational data.

 $736\ 00:40:09.410$  --> 00:40:13.763 And we can estimate and compare DTRs along a continuum.

 $737\ 00:40:14.900 \longrightarrow 00:40:17.390$  We can formulate a causal model

738 00:40:17.390 --> 00:40:22.050 for the smooth effect of Q on the task quantile of XQ.

739 00:40:22.050 --> 00:40:25.380 This is our composite outcome with separate parameters

740 00:40:25.380 --> 00:40:29.030 capturing the effect of treat immediately,

 $741\ 00:40:29.030 \longrightarrow 00:40:31.320$  and the effect of never treat.

 $742\ 00{:}40{:}31.320 \dashrightarrow 00{:}40{:}35.360$  And then we can parametrize the model using splines of Q

 $743\ 00:40:35.360 \longrightarrow 00:40:39.853$  for the third term here, to gain statistical efficiency.

744 00:40:41.360 --> 00:40:46.360 And we can obtain a consistent estimator of effect of Q

 $745\ 00:40:46.940 \longrightarrow 00:40:50.010$  by solving the weighted quantile regression

 $746\ 00:40:50.010 \longrightarrow 00:40:51.383$  estimating equation.

 $747\ 00:40:52.820 \longrightarrow 00:40:55.193$  So what should be the weights?

 $748\ 00:40:57.020 \longrightarrow 00:40:59.410$  First, we assume there's no dropout or death

749 00:40:59.410 --> 00:41:01.739 prior to the target measurement time.

 $750\ 00:41:01.739 \longrightarrow 00:41:06.739$  In the discrete time setting with common time point,

751 00:41:06.930  $\rightarrow$  00:41:09.900 the form of the weights have already been done

 $752\ 00:41:09.900 --> 00:41:13.570$  It has been derived in several papers.

753 00:41:13.570 --> 00:41:16.680 Essentially, the denominator of the weight

754 00:41:16.680 --> 00:41:18.980 is this conditional probability.

 $755\ 00{:}41{:}18.980 \dashrightarrow 00{:}41{:}22.022$  It's a conditional probability of the person being compliant

 $756\ 00:41:22.022 \longrightarrow 00:41:27.022$  all throughout the follow up, given the covariate history.

 $757\ 00:41:29.010 --> 00:41:34.010$  So if we have a common set of discrete time points,

 $758\ 00:41:34.160 \longrightarrow 00:41:37.640$  it's a cumulative a product of the conditional probability

759 00:41:37.640 --> 00:41:42.150 of this person being compliant at every time point.

760 00:41:42.150 --> 00:41:45.740 And essentially if the rule says treat,

 $761\ 00:41:45.740 --> 00:41:48.350$  it's a condition of probability of the person

762 00:41:48.350 --> 00:41:51.240 actually being treated at this time point,

763 00:41:51.240 --> 00:41:53.450 if the rule says not treat,

764~00:41:53.450 --> 00:41:56.850 as a conditional probability of this person not treated

 $765\ 00:41:56.850 \longrightarrow 00:41:58.320$  by this time point.

766 00:41:58.320 --> 00:42:02.140 So in order to estimate this probability,

 $767\ 00:42:02.140 \longrightarrow 00:42:04.170$  we just need to model the observed

 $768\ 00:42:04.170 --> 00:42:07.613$  treatment initiation process among those regimen compliers,

769 00:42:09.360 --> 00:42:11.550 but this is for discrete time setting.

770 00:42:11.550 --> 00:42:14.380 What would be the continuous time weights?

771 00:42:14.380 --> 00:42:17.900 We note that the occurrence of treatment initiation

772 00:42:17.900 --> 00:42:21.170 in a small time interval T and T plus TD

773 00:42:21.170 --> 00:42:26.170 is actually a Bernoulli trial with outcome DNA of T.

 $774\ 00:42:27.690 \longrightarrow 00:42:31.150$  So then we can rewrite this probability,

 $775\ 00:42:31.150 \longrightarrow 00:42:32.950$  this probability here,

776 00:42:32.950 --> 00:42:36.850 in the form of individual partial likelihood

 $777\ 00:42:36.850 \longrightarrow 00:42:38.840$  for the counting process of A.

 $778\ 00:42:40.070 \longrightarrow 00:42:44.680$  And now we note that when DT becomes smaller and smaller,

779 00:42:44.680 --> 00:42:49.640 this finite product approaches a product integral.

 $780\ 00:42:49.640 --> 00:42:54.390$  So then this finite product can be rewritten

 $781~00{:}42{:}54.390 {\: -->\:} 00{:}42{:}58.490$  as a final product over jump times of the counting process

 $782\ 00:42:58.490 \longrightarrow 00:43:01.900$  for A times the survival function.

783 00:43:01.900 --> 00:43:04.970 And then by recognizing that each individual

784 00:43:04.970 --> 00:43:09.730 had at most one jump at exactly AI.

 $785\ 00:43:09.730 --> 00:43:14.563$  Now we can further reduce this probability to this form.

 $786\ 00:43:15.480 --> 00:43:18.100$  Which suggests weighting scheme.

 $787~00{:}43{:}18.100 \dashrightarrow 00{:}43{:}22.800$  Essentially it says for those who have been treated

788 00:43:22.800 --> 00:43:25.170 by a T star, we would weight them

789 00:43:25.170 --> 00:43:28.370 by the conditional density function of A.

790 00:43:28.370  $\rightarrow$  00:43:32.040 For those who haven't been treated by the time T star,

791  $00:43:32.040 \longrightarrow 00:43:36.140$  we would weight them by the survival or function of A.

792 00:43:36.140 --> 00:43:38.760 So if you recall the weighting scheme

 $793\ 00:43:38.760 --> 00:43:42.580$  for the first motivating example, this is exactly the same,

 $794\ 00:43:42.580 \longrightarrow 00:43:43.893$  the same rating scheme,

 $795\ 00:43:44.930 \longrightarrow 00:43:46.790$  but we took different approaches.

 $796\ 00:43:46.790 \longrightarrow 00:43:48.270$  The first example,

797 00:43:48.270 --> 00:43:50.610 we use a random Aalen derivatives

 $798\ 00:43:50.610 \longrightarrow 00:43:52.350$  to derive the weighting scheme.

 $799\ 00:43:52.350 --> 00:43:56.187$  The second project we derive the limit

800 00:43:57.930 --> 00:44:00.120 of the finite product,

801 00:44:00.120 --> 00:44:02.330 but using different approaches,

 $802\ 00:44:02.330 \longrightarrow 00:44:04.343$  we arrive at the same weighting scheme.

 $803\ 00{:}44{:}05.750 \dashrightarrow 00{:}44{:}10.130$  And so similarly we modeled the intensity process

 $804\ 00:44:10.130 \longrightarrow 00:44:11.710$  of treatment initiation.

 $805\ 00:44:11.710 \longrightarrow 00:44:13.193$  We estimate the weights.

- $806\ 00:44:15.480 \longrightarrow 00:44:17.740$  So if there was a censoring or death
- 807 00:44:17.740 --> 00:44:19.810 prior to target measurement time,
- $808\ 00:44:19.810 --> 00:44:22.810$  we would have to assume once lost to follow up
- 809 00:44:22.810 --> 00:44:25.000 at a time prior to T star,
- $810~00{:}44{:}25.000 \dashrightarrow 00{:}44{:}28.090$  the treatment and regimen status remain constant.
- $811\ 00:44:28.090 --> 00:44:30.620$  And this way we will just estimate the weights
- 812 00:44:30.620 --> 00:44:35.620 up to a time point CI, and if the person died before T star,
- $813~00{:}44{:}36.530 \dashrightarrow 00{:}44{:}39.920$  then we would only evaluate compliance
- $814\ 00{:}44{:}39.920 --> 00{:}44{:}43.283$  and treatment initiation processes up to time TI.
- 815 00:44:45.390 --> 00:44:48.590 Okay, so for missing outcomes,
- $816\ 00:44:48.590 \longrightarrow 00:44:51.660$  we propose a joint modeling approach.
- $817\ 00{:}44{:}51.660 \dashrightarrow 00{:}44{:}56.150$  We specify a two-level model for the observed CD4 process.
- 818 00:44:56.150 --> 00:44:57.020 The first level,
- $819\ 00{:}44{:}57.020 \dashrightarrow 00{:}45{:}01.690$  the observed CD4 process is a true CD4 trajectory
- $820\ 00:45:01.690 \longrightarrow 00:45:03.570$  plus some arrow process.
- 821 00:45:03.570 --> 00:45:04.720 The second level,
- 822 00:45:04.720 --> 00:45:07.020 we relate the true CD4 trajectory
- $823\ 00:45:07.020 \longrightarrow 00:45:12.020$  to baseline characteristics and treatment initiation time,
- 824 00:45:13.170 --> 00:45:16.440 and some subject specific random effects,
- 825 00:45:16.440 --> 00:45:18.900 capturing subject-specific deviations
- $826\ 00:45:18.900 \longrightarrow 00:45:22.390$  from the mean trajectories.
- 827 00:45:22.390 --> 00:45:25.610 And now we propose a hazard model for deaths
- $828\ 00:45:25.610 --> 00:45:30.270$  uses the true CD4 trajectory as a covariate
- $829\ 00:45:30.270 \longrightarrow 00:45:32.970$  linking the two processes,
- $830~00{:}45{:}32.970 \dashrightarrow 00{:}45{:}37.580$  Linking the death process and linking with a CD4 process.

- 831  $00:45:37.580 \longrightarrow 00:45:39.326$  Now we use the joint model
- 832 00:45:39.326 --> 00:45:42.730 to impute the missing outcomes
- $833\ 00{:}45{:}42.730 \dashrightarrow 00{:}45{:}45.580$  and estimate the variance of the target estimator
- 834 00:45:45.580 --> 00:45:47.580 using Rubin's combination wall.
- $835\ 00{:}45{:}49.530 \dashrightarrow 00{:}45{:}53.650$  So we applied this method to the IeDEA dataset.
- 836 00:45:53.650 --> 00:45:58.650 IeDEA is another HIV consortium based in West Kenya.
- 837 00:46:00.230 --> 00:46:02.920 So we have almost 2000 data.
- $838\ 00{:}46{:}02.920 \dashrightarrow 00{:}46{:}06.910$  We see that the CD4 is pretty sparsely measured
- $839\ 00:46:06.910 \longrightarrow 00:46:11.910$  and death rate is low around three and 4%.
- 840 00:46:11.960 --> 00:46:15.410 Most of patients have been treated by one year
- $841\ 00:46:16.480 \longrightarrow 00:46:20.660$  and we have a set of covariates.
- $842\ 00:46:20.660$  --> 00:46:23.993 Some of them are time varying, some of them are time fixed.
- 843 00:46:25.710 --> 00:46:29.550 We proposed three target estimators,
- $844\ 00:46:29.550 \longrightarrow 00:46:33.890$  so first we're interested in mortality proportion.
- $845\ 00{:}46{:}33.890$  -->  $00{:}46{:}37.870$  We're also interested in the median of the distribution
- $846\ 00:46:37.870 \longrightarrow 00:46:40.631$  of the composite outcome XQ.
- 847 00:46:40.631 --> 00:46:44.600 We also looked at CD4 among survivors,
- $848\ 00{:}46{:}44.600 {\: -->\:} 00{:}46{:}49.160$  but this estimator does not have a causal interpretation
- 849 00:46:49.160 --> 00:46:53.320 because it conditions on having survived two T star.
- 850 00:46:53.320 --> 00:46:55.350 So it only measures association,
- $851\ 00{:}46{:}55{.}350 \dashrightarrow 00{:}47{:}00{.}350$  but the first two estimators have causal interpretations.
- $852~00{:}47{:}02.830 \dashrightarrow 00{:}47{:}05.060$  So we first look at the effectiveness
- $853\ 00{:}47{:}05.060 \dashrightarrow 00{:}47{:}09.960$  of five specific regimens for both one year and two years

- $854\ 00:47:09.960 \longrightarrow 00:47:11.698$  after diagnosis.
- $855\ 00:47:11.698 --> 00:47:16.660$  We can see that the immediate treatment initiation
- 856 00:47:18.270 --> 00:47:21.200 lead to significant lower mortality rate
- 857 00:47:21.200 --> 00:47:24.050 and significantly higher median values
- $858\ 00:47:24.050 --> 00:47:29.050$  of the composite alcohol compared to delayed treatment.
- $859\ 00:47:29.570 \longrightarrow 00:47:32.460$  And the never treat initiation
- $860\ 00:47:32.460 --> 00:47:36.800$  will lead to a significantly higher mortality probability.
- 861 00:47:36.800 --> 00:47:41.440 And for those who do survive to T star,
- 862 00:47:41.440 --> 00:47:44.340 their CD4 count is higher.
- $863~00{:}47{:}44.340 \dashrightarrow 00{:}47{:}48.290$  So resulting higher to theta Q2 and higher theta Q3
- $864\ 00:47:48.290 \longrightarrow 00:47:53.290$  compared to other delayed treatment regimen.
- $865\ 00:47:53.680 --> 00:47:57.430$  So this may suggest that those who do survive
- 866 00:47:57.430 --> 00:47:59.670 to T-star without any treatment,
- $867\ 00:47:59.670 \longrightarrow 00:48:01.900$  maybe they are relatively healthier
- $868\ 00:48:01.900 \longrightarrow 00:48:03.593$  at the beginning of the followup.
- $869\ 00{:}48{:}05.290 \dashrightarrow 00{:}48{:}10.000$  Okay, and then we also plot the dose response curve
- $870\ 00:48:10.000 \longrightarrow 00:48:14.350$  of the median value of the composite outcome
- 871 00:48:14.350  $\rightarrow$  00:48:17.250 versus DTR Q,
- 872 00:48:17.250 --> 00:48:22.250 also suggests that the immediate treatment
- $873\ 00{:}48{:}22.320 \dashrightarrow 00{:}48{:}26.920$  would lead to significantly higher median values of XQ,
- $874\ 00:48:26.920 \longrightarrow 00:48:28.530$  and also as illustration
- $875\ 00:48:28.530 --> 00:48:30.480$  of the gained statistical efficiency
- $876~00{:}48{:}30.480 \to 00{:}48{:}35.480$  by modeling the smooth effect Q on the quantile of the XQ.
- $877\ 00:48:36.950 --> 00:48:39.880$  The variance in the one year outcome
- 878~00:48:39.880 --> 00:48:44.880 associated with Q equals 350, achieved about 15% reduction

 $879\ 00:48:45.860 --> 00:48:49.453$  compared to that from the regimen specific estimates.

880  $00:48:50.970 \longrightarrow 00:48:55.164$  So we gain a bit of our statistical efficiency

 $881\ 00:48:55.164 \longrightarrow 00:48:58.313$  by modeling the smooth effect.

 $882\ 00:49:00.000 \longrightarrow 00:49:02.390$  So there are several strands of continuous time

883  $00:49:02.390 \longrightarrow 00:49:03.940$  marginal structure model.

 $884\ 00:49:03.940 \longrightarrow 00:49:07.850$  We see that we can derive, using different approaches,

 $885\ 00:49:07.850$  --> 00:49:11.598 closed form of weights for continuous-time treatment.

 $886\ 00:49:11.598 --> 00:49:16.052$  It can handle complex dataset on its own terms

 $887\ 00:49:16.052 --> 00:49:20.100$  without having to artificially align measurement times,

 $888\ 00:49:20.100 \dashrightarrow 00:49:22.943$  which could possibly lead to loss of information.

889 00:49:23.840 --> 00:49:26.560 It is amenable to many different outcomes.

890 00:49:26.560 --> 00:49:28.260 We've used the survival outcomes,

 $891\ 00:49:28.260 \longrightarrow 00:49:30.163$  we've used composite outcomes.

892 00:49:31.280 --> 00:49:34.340 You can also handle many data complications

893 00:49:34.340 --> 00:49:37.150 introduced by various censoring patterns

 $894\ 00:49:37.150 --> 00:49:39.503$  within the same marginal structure model.

 $895\ 00:49:40.650 \longrightarrow 00:49:42.748$  So these are the strengths,

 $896\ 00{:}49{:}42.748 \dashrightarrow 00{:}49{:}46.150$  but there are also limitations with this approach of course.

897 00:49:46.150 --> 00:49:49.970 One notable limitation is extreme ways,

 $898~00{:}49{:}49.970 \dashrightarrow 00{:}49{:}53.023$  which could possibly lead to unstable estimates.

899 00:49:54.290 --> 00:49:56.840 So how to address this issue,

900 00:49:56.840 --> 00:50:00.450 especially for time varying confounding

901 00:50:00.450 --> 00:50:04.330 with censored outcome, this would be a challenging task,

902 00:50:04.330 --> 00:50:06.150 but if we can solve this issue,

- 903 00:50:06.150 --> 00:50:09.770 it might be a very important contribution to the field.
- $904\ 00:50:09.770 \longrightarrow 00:50:13.840$  So this is something my colleagues and I
- $905\ 00:50:13.840 \longrightarrow 00:50:18.396$  have been thinking about and working on for some time.
- $906\ 00:50:18.396 \longrightarrow 00:50:21.980$  Another limitation is that we know
- $907\ 00:50:21.980 \longrightarrow 00:50:25.060$  that weighting-based estimator is less efficient
- $908\ 00:50:25.060 \longrightarrow 00:50:27.620$  than the so-called G methods.
- 909 00:50:27.620 --> 00:50:30.020 The G computation, G estimation,
- 910 00:50:30.020 --> 00:50:32.470 and both G methods require integrating
- 911  $00:50:32.470 \longrightarrow 00:50:35.250$  over the space of longitudinal confounders.
- 912 00:50:35.250 --> 00:50:38.270 So the G methods are computationally
- 913 00:50:38.270 --> 00:50:40.160 much, much more expensive
- 914 00:50:40.160 --> 00:50:43.960 than the marginal structure model-based methods.
- 915 00:50:43.960 --> 00:50:46.603 And as far as I know,
- 916 00:50:47.849 --> 00:50:50.130 currently there's no continuous time version
- $917\ 00:50:50.130 \longrightarrow 00:50:52.560$  of the G computation methods.
- 918 00:50:52.560 --> 00:50:56.300 Judith Lok has a paper, back in 2008.
- 919 00:50:56.300 --> 00:50:59.980 She developed theory for continuous time Gestimation,
- 920 00:50:59.980 --> 00:51:03.470 but I have yet to see a practical implementation
- 921 00:51:03.470 --> 00:51:04.910 of this method.
- 922 00:51:04.910 --> 00:51:09.910 So this could be another avenue for future research,
- 923 00:51:11.000 --> 00:51:14.940 how to increase efficiency of the continuous time
- $924\ 00:51:14.940 \longrightarrow 00:51:16.433$  weighting-based methods.
- $925\ 00:51:17.650 \longrightarrow 00:51:20.713$  And here's some key references.
- 926 00:51:21.744 --> 00:51:23.786 Thank you.
- 927 00:51:23.786 --> 00:51:25.450 Thank you Liangyuan for this very interesting

- $928\ 00:51:25.450 \longrightarrow 00:51:29.250$  and comprehensive presentation.
- 929 00:51:29.250  $\rightarrow$  00:51:32.460 Let's see if we have any questions from the audience.
- 930  $00:51:32.460 \longrightarrow 00:51:33.500$  If there's any questions,
- $931\ 00:51:33.500 \longrightarrow 00:51:36.450$  please feel free to unmute yourself and speak
- 932  $00:51:36.450 \longrightarrow 00:51:38.393$  or type in the chat.
- 933 00:51:43.010 --> 00:51:45.040 [Donna] Thanks, it was a very interesting talk.
- 934 00:51:45.040 --> 00:51:47.300 This is Donna Spiegelman.
- 935 00:51:47.300 --> 00:51:48.474 Hi, Donna.
- 936 00:51:48.474 --> 00:51:49.307 Yeah, hi.
- 937 00:51:49.307 --> 00:51:50.910 I was wondering I might've missed it,
- 938  $00:51:50.910 \longrightarrow 00:51:55.160$  but did you say much about estimating the variance?
- 939 00:51:55.160 --> 00:51:58.260 I see you have (indistinct) around the curve,
- 940 00:51:58.260 --> 00:52:01.479 so you must derive the variance.
- 941 00:52:01.479 --> 00:52:03.420 So I'm wondering if you could say a little bit about that
- $942\ 00:52:03.420 \longrightarrow 00:52:04.640$  or a little more about that
- 943 00:52:04.640 --> 00:52:07.350 if I missed what you did say.
- 944 00:52:07.350 --> 00:52:08.680 Sure, sure, sure.
- 945 00:52:08.680 --> 00:52:11.520 So for this one, this is the second example,
- $946\ 00:52:11.520 --> 00:52:14.600$  for this one we have multiple amputation
- 947 00:52:14.600 --> 00:52:16.133 and we also have weighting.
- 948 00:52:20.045 --> 00:52:21.195 So with weighting part,
- 949 00:52:22.080  $\rightarrow$  00:52:26.160 the variance was estimated using bootstrap
- $950\ 00:52:26.160$  --> 00:52:29.310 for multiple amputation, and then we combined,
- 951 00:52:29.310 --> 00:52:32.713 so it's a bootstrap nested within multiple imputation.
- $952\ 00:52:32.713 --> 00:52:35.070$  So then we use the Rubin's combination role
- $953\ 00:52:35.070 \longrightarrow 00:52:37.093$  to estimate the total variance.

 $954\ 00:52:38.150 \longrightarrow 00:52:43.150$  For the first example, we actually used a bootstrap,

955 00:52:45.210 --> 00:52:49.760 and the coverage probability was actually okay.

956  $00:52:49.760 \longrightarrow 00:52:51.473$  It's good for the estimator.

957 00:52:52.340 --> 00:52:55.233 - Did you think about asymptotic variants derivations?

958 00:52:56.190 --> 00:52:57.023 - I did.

959 00:52:58.170 --> 00:53:00.023 It was a very difficult task,

960 00:53:02.556 --> 00:53:05.200 there's a story about our first paper

 $961\ 00:53:05.200 \longrightarrow 00:53:06.423$  found that about it.

962 00:53:09.940 --> 00:53:12.120 It was first submitted to Jaza

963 00:53:12.120 --> 00:53:16.370 and then they asked about the asymptotic variants

 $964\ 00:53:16.370 \longrightarrow 00:53:17.520$  about the estimator.

965 00:53:17.520  $\rightarrow$  00:53:21.830 And it's quite complex because they involve the splice

 $966\ 00:53:21.830 \longrightarrow 00:53:25.340$  and involves the survival data.

967  $00:53:25.340 \longrightarrow 00:53:28.773$  And we have already approved as a consistency,

968 00:53:33.706 --> 00:53:36.539 and it also involves optimization.

969  $00:53:38.340 \longrightarrow 00:53:40.260$  So it's just comes to-

970 00:53:40.260 --> 00:53:42.770 - What's the optimization piece.

971 00:53:42.770 --> 00:53:47.060 - Oh, it's the model based optimal treatment initiation time

 $972\ 00:53:47.060 \longrightarrow 00:53:48.780$  that will lead to the maximum survival

973 00:53:48.780 --> 00:53:53.780 at predefined time points.

974 00:53:53.930 --> 00:53:57.630 Right, so they are interested in the optimization.

 $975\ 00:53:57.630 \longrightarrow 00:54:00.230$  So the inference about the optimized

976 00:54:00.230 --> 00:54:02.410 treatment initiation time.

 $977\ 00:54:02.410 \longrightarrow 00:54:04.320$  We did some empirical evidence

978 00:54:04.320 --> 00:54:08.217 for like the largest sample convergence rate,

979 00:54:08.217 --> 00:54:13.217 but we weren't successful at deriving asymptotic variants.

980 00:54:14.450 --> 00:54:18.170 So that's another piece, I think maybe,

981 00:54:18.170 --> 00:54:19.003 I don't know.

 $982\ 00:54:19.003 --> 00:54:21.280$  We had this discussion among colleagues

983 00:54:21.280 --> 00:54:23.720 and also my advisor at the time,

984 00:54:23.720  $\rightarrow$  00:54:27.840 we just not sure about whether it's worth the effort

 $985\ 00:54:27.840 \longrightarrow 00:54:29.573$  to go and do that route.

986 00:54:30.420 --> 00:54:32.220 - It's probably way more complex

 $987\ 00:54:32.220 \longrightarrow 00:54:34.130$  than just the usual derivation.

988 00:54:34.130 --> 00:54:36.470 'Cause you do have like two weighting models,

989 00:54:36.470 --> 00:54:39.590 which are also survival models,

 $990\ 00:54:39.590 \longrightarrow 00:54:41.900$  and also the derivation that these variances

991 00:54:41.900 --> 00:54:44.440 sometimes can be specific to the choice

992  $00:54:44.440 \longrightarrow 00:54:45.800$  of these (indistinct) models.

 $993~00:54:45.800 \rightarrow 00:54:48.590$  And so if you have a variance and the cup's model,

994 00:54:48.590 --> 00:54:51.696 it does not apply to other forms of models,

995 00:54:51.696 --> 00:54:54.290 I guess it's really a trade-off right?

996  $00:54:54.290 \longrightarrow 00:54:57.343$  - Yeah, it is a trade off.

997 00:54:58.420 --> 00:55:03.330 It's still an open question and nobody had done it yet,

998 00:55:03.330  $\rightarrow$  00:55:06.240 but just, whether you're thinking it's was the effort

999 00:55:06.240 --> 00:55:10.020 just to devote a couple of years to work on that

1000~00:55:10.020 --> 00:55:14.780 - So was bootstrap time consuming for these datasets,

 $1001\ 00:55:14.780 --> 00:55:18.211$  for this data analysis, or they're pretty manageable.

 $1002\ 00:55:18.211 --> 00:55:19.780$  - They're pretty manageable.

 $1003\ 00:55:19.780 --> 00:55:23.070$  And it looks complicated because we have to weight everybody

- $1004\ 00:55:23.070 \longrightarrow 00:55:24.110$  that had event.
- $1005\ 00{:}55{:}24.110 --> 00{:}55{:}27.260$  We also have to weight everywhere in the risk set
- $1006\ 00:55:27.260 \longrightarrow 00:55:28.350$  at any time point.
- $1007\ 00:55:28.350 --> 00:55:31.710$  So it looks pretty complex, but still manageable.
- $1008\ 00:55:34.840 --> 00:55:38.000$  Another reason is because we use parametric models.
- $1009\ 00:55:38.000 \longrightarrow 00:55:41.220$  If we wanted to,
- $1010~00:55:41.220 \dashrightarrow 00:55:45.810$  I'm not aware of any machine learning algorithm
- 1011 00:55:45.810 --> 00:55:48.240 that can handle survival data,
- 1012 00:55:48.240 --> 00:55:50.543 but also with time varying covariates,
- $1013\ 00:55:51.930 \longrightarrow 00:55:54.260$  that's something I'm also thinking about.
- $1014\ 00:55:54.260 \longrightarrow 00:55:56.210$  Like, if we use those algorithm
- 1015 00:55:56.210 --> 00:55:58.550 might be more time consuming,
- 1016~00:55:58.550 --> 00:56:02.640 but with just a parametric models, it's pretty manageable.
- 1017 00:56:02.640 --> 00:56:03.540 And when you're bootstrapped,
- 1018 00:56:03.540 --> 00:56:05.810 you go back to the weight models
- 1019 00:56:05.810 --> 00:56:08.340 and refit the weight models every time?
- $1020\ 00:56:08.340 \longrightarrow 00:56:09.615$  Yeah.
- $1021\ 00:56:09.615 \longrightarrow 00:56:12.430$  But the variable is pre-determined.
- $1022\ 00:56:12.430 --> 00:56:14.940$  So that's what you mentioned, machine learning.
- $1023\ 00:56:14.940 \longrightarrow 00:56:17.120$  So the variables are predetermined
- $1024\ 00:56:17.120 \longrightarrow 00:56:19.080$  and they're functional forms in the model,
- $1025\ 00:56:19.080 \longrightarrow 00:56:21.960$  but the coefficients that correspond to them
- $1026\ 00:56:21.960 \longrightarrow 00:56:24.400$  are re estimated for each bootstrap.
- $1027\ 00:56:24.400 \longrightarrow 00:56:25.720$  Very estimated.
- 1028 00:56:25.720 --> 00:56:27.046 Right, right, right.
- $1029\ 00:56:27.046 \longrightarrow 00:56:27.879$  Exactly.
- $1030\ 00:56:27.879 \longrightarrow 00:56:28.712\ Yeah.$

- $1031\ 00:56:28.712 --> 00:56:30.090$  Great question.
- $1032\ 00:56:30.090 \longrightarrow 00:56:31.070$  Yeah.
- $1033\ 00:56:31.070 \longrightarrow 00:56:33.063$  So a lot of open questions still.
- 1034 00:56:34.900 --> 00:56:38.083 So any other questions from the audience?
- $1035\ 00:56:41.290 \longrightarrow 00:56:42.623$  I have another comment.
- 1036 00:56:43.900 --> 00:56:46.810 So by getting back to this,
- $1037~00{:}56{:}46.810 \dashrightarrow 00{:}56{:}48.870$  that you re estimated the coefficients
- $1038\ 00:56:48.870 \longrightarrow 00:56:50.440$  for the weight models.
- $1039\ 00:56:50.440 --> 00:56:54.053$  So in sort of the standard marginal structural model,
- $1040\ 00{:}56{:}54.053 --> 00{:}56{:}58.800$  the variability due to those weight models is ignored.
- $1041\ 00:56:58.800 \longrightarrow 00:57:00.780$  And the robust variance is used
- 1042 00:57:00.780 --> 00:57:02.430 and said to be an overestimate,
- $1043\ 00:57:02.430 \longrightarrow 00:57:06.350$  implying that if you took that variation into account,
- $1044\ 00:57:06.350 \longrightarrow 00:57:08.450$  you'd get a smaller variance
- $1045\ 00:57:08.450 \longrightarrow 00:57:11.290$  and you might see the same thing here with your bootstraps.
- 1046 00:57:11.290 --> 00:57:14.508 If you took the weight models as fixed,
- $1047\ 00{:}57{:}14.508 --> 00{:}57{:}17.990$  you might find that you have a less efficient estimator,
- 1048 00:57:17.990 --> 00:57:19.760 which is kind of interesting
- $1049\ 00:57:19.760 --> 00:57:23.150$  just in terms of say a methods paper to show,
- $1050\ 00{:}57{:}23.150 {\: -->\:} 00{:}57{:}26.060$  because there's different ways to do bootstraps,
- $1051\ 00{:}57{:}26.060 {\ \mbox{--}>}\ 00{:}57{:}30.120$  but here you're automatically taking the estimation
- 1052 00:57:30.120 --> 00:57:31.680 of the weight models into account,
- $1053\ 00:57:31.680 --> 00:57:35.300$  which is not saying that say the classic paper
- $1054\ 00:57:35.300 \longrightarrow 00:57:38.114$  by Hernan in epidemiology,
- $1055\ 00{:}57{:}38.114$  -->  $00{:}57{:}42.310$  that's ignored and the robust variance is recommended.

- $1056\ 00:57:42.310 \longrightarrow 00:57:43.460 Hmm.$
- $1057\ 00:57:43.460 \longrightarrow 00:57:46.090$  It's a very great comment.
- $1058\ 00:57:46.090 --> 00:57:47.480$  Something I have to think about.
- $1059~00:57:47.480 \longrightarrow 00:57:51.860$  So you're saying that in each bootstrap,
- $1060\ 00:57:51.860 \longrightarrow 00:57:54.810$  when we estimate the weight model, we fix the weight model.
- $1061\ 00:57:56.903 --> 00:57:59.763$  So the coefficients from the weight model stay fixed-
- $1062\ 00:58:00.850 --> 00:58:02.810$  Yeah, so you don't even do a bootstrap for that.
- $1063\ 00:58:02.810$  --> 00:58:06.465 You basically hold the weight model as a constant,
- $1064\ 00:58:06.465 \longrightarrow 00:58:07.298$  and then you'd-
- $1065\ 00:58:07.298 \longrightarrow 00:58:08.570$  Robust variance.
- 1066 00:58:08.570 --> 00:58:10.950 Yeah, you use the robust variance,
- 1067 00:58:10.950 --> 00:58:12.690 which I guess it's a little tricky
- $1068~00{:}58{:}12.690 \dashrightarrow 00{:}58{:}14.520$  because now you don't have the robust variance
- 1069 00:58:14.520 --> 00:58:15.776 because you're not using it,
- $1070\ 00:58:15.776$  --> 00:58:20.776 but it seems the bootstrap analog of the approach taken
- 1071 00:58:20.870 --> 00:58:24.230 would be to just fit the weight model once,
- 1072 00:58:24.230 --> 00:58:26.870 treat that fixed unknown,
- $1073\ 00{:}58{:}26.870$  -->  $00{:}58{:}31.190$  and then only bootstrap on the outcome model.
- 1074 00:58:31.190 --> 00:58:32.260 Right, right.
- $1075\ 00{:}58{:}32.260 --> 00{:}58{:}33.093\ \mathrm{Yeah}.$
- $1076\ 00:58:33.093 --> 00:58:34.618$  [Fan Li] Totally. Yeah.
- $1077\ 00:58:34.618 \longrightarrow 00:58:36.170$  Interesting.
- $1078\ 00:58:36.170 \longrightarrow 00:58:37.753$  Take that in as a note.
- $1079\ 00:58:39.300 \longrightarrow 00:58:41.590$  So I do have a question as well.
- $1080~00{:}58{:}41.590 \dashrightarrow 00{:}58{:}44.200$  I think Liangyuan you had presented two applications

- 1081 00:58:44.200 --> 00:58:46.970 at the HIV observational studies,
- $1082\ 00{:}58{:}46.970 \dashrightarrow 00{:}58{:}51.150$  do you see the application that these new methods
- $1083\ 00:58:51.150 \longrightarrow 00:58:53.580$  to other areas as well
- $1084\ 00:58:54.450 \longrightarrow 00:58:57.060$  to solve the other questions? Yeah.
- $1085~00{:}58{:}57.060 \dashrightarrow 00{:}59{:}01.770$  Yeah, actually this is not pertaining to HIV area.
- 1086 00:59:01.770 --> 00:59:06.140 It's actually in the public health areas.
- $1087\ 00:59:06.140 --> 00:59:09.623$  A lot of questions are involving
- $1088\ 00:59:13.430 --> 00:59:15.580$  this statistical formulation.
- 1089 00:59:15.580 --> 00:59:16.693 So for example,
- $1090\ 00:59:17.600 \longrightarrow 00:59:22.600$  I've been collaborating with an epidemiologist at Columbia.
- $1091\ 00:59:22.623 --> 00:59:26.540$  They are doing cardiovascular research.
- $1092\ 00:59:26.540 \longrightarrow 00:59:29.223$  So one research question is that,
- $1093\ 00:59:30.930$  --> 00:59:35.930 I think it's blood pressure lowering intervention.
- $1094\ 00:59:36.690 \longrightarrow 00:59:40.470$  So blood lowering innovation is very useful
- 1095 00:59:40.470 --> 00:59:42.623 for preventing cardiovascular diseases,
- 1096 00:59:45.460 --> 00:59:46.850 but they don't know.
- $1097\ 00:59:46.850 --> 00:59:49.630$  And there also a lack of randomized control trials.
- $1098\ 00:59:49.630 \longrightarrow 00:59:52.920$  What is the optimal threshold
- $1099\ 00:59:52.920 \longrightarrow 00:59:57.200$  to start giving the blood lowering treatment?
- $1100\ 00:59:57.200 \longrightarrow 00:59:59.100$  So this is exactly the same form
- $1101\ 00:59:59.100 \longrightarrow 01:00:01.410$  as our second motivating example.
- $1102\ 01:00:01.410 --> 01:00:04.350$  Like what is the optimal CD4 threshold
- 1103 01:00:04.350 --> 01:00:06.250 to start the HIV treatment?
- $1104\ 01{:}00{:}06.250 \dashrightarrow 01{:}00{:}08.860$  And their question is what is the optimal threshold
- $1105\ 01:00:08.860 \longrightarrow 01:00:12.650$  to start the blood lowering treatment?
- $1106\ 01:00:12.650 \longrightarrow 01:00:17.070$  So I think there's a lot of possibility
- $1107\ 01:00:19.780 \longrightarrow 01:00:21.890$  as to apply these kinds of methods

- $1108\ 01:00:21.890 \longrightarrow 01:00:24.240$  in other health research area.
- 1109 01:00:24.240 --> 01:00:26.320 Yeah, it's a huge controversy
- 1110 01:00:26.320 --> 01:00:28.310 in terms of the treatment of hypertension,
- $1111\ 01:00:28.310 \longrightarrow 01:00:30.520$  what's the optimal blood pressure
- $1112\ 01:00:30.520 \longrightarrow 01:00:33.147$  to start antihypertensives.
- 1113 01:00:33.147 --> 01:00:35.310 And I think there was a very large trial
- 1114 01:00:35.310 --> 01:00:37.740 that showed that it was better to start it
- $1115\ 01:00:37.740 \longrightarrow 01:00:42.120$  at a much earlier threshold than what current practices.
- $1116\ 01:00:42.120 --> 01:00:46.710$  And it's very troublesome for people around the world
- $1117\ 01:00:46.710 \longrightarrow 01:00:49.170$  because these medicines are expensive.
- 1118 01:00:49.170 --> 01:00:50.780 And if you see now,
- $1119\ 01:00:50.780 \longrightarrow 01:00:54.060$  like another like 40% of the population
- $1120\ 01:00:54.060 --> 01:00:58.180$  should now be initiated a antihypertensive medication,
- $1121\ 01:00:58.180 \longrightarrow 01:01:01.090$  well, most countries can't even afford that.
- $1122\ 01:01:01.090 --> 01:01:04.730$  So the implications of these different thresholds
- $1123\ 01{:}01{:}04.730 {\: -->\:} 01{:}01{:}08.630$  is a very big topic of sort of substantive research
- $1124\ 01:01:08.630 \longrightarrow 01:01:10.240$  and debate right now.
- 1125 01:01:10.240 --> 01:01:11.720 Well, that's great to know,
- 1126 01:01:11.720 --> 01:01:14.365 there's urgent need for that.
- 1127 01:01:14.365 --> 01:01:16.090 (indistinct)
- $1128\ 01:01:16.090 \longrightarrow 01:01:17.040$  Totally.
- $1129\ 01:01:17.040 \longrightarrow 01:01:19.420$  All right, I think we are at the hour,
- $1130\ 01{:}01{:}19.420 {\: \hbox{--}}{\:\raisebox{-2pt}{$>$}}\ 01{:}01{:}24.420$  so thanks Liangyuan again for your great presentation
- 1131 01:01:25.020 --> 01:01:27.600 and if the audience has any questions,
- $1132\ 01{:}01{:}27.600 --> 01{:}01{:}30.610$  I'm sure Liangyuan is happy to take any questions offline
- $1133\ 01:01:30.610 \longrightarrow 01:01:31.623$  by emails.

 $1134\ 01{:}01{:}32.570 \dashrightarrow 01{:}01{:}37.570$  And I think this is the final seminar of our fall series,

1135 01:01:37.710 --> 01:01:40.430 and I hope to see everyone next spring,

1136 01:01:40.430 --> 01:01:42.040 have a good holiday.

1137 01:01:42.040 --> 01:01:43.000 Thank you.

1138 01:01:43.000 --> 01:01:44.140 - Thank you.

1139 01:01:44.140 --> 01:01:45.050 - Bye. - Bye.